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Equal relation between the extra connectivity and pessimistic diagnosability for some regular graphs

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ABSTRACT

Extra connectivity and the pessimistic diagnosis are two crucial subjects for a multiprocessor system's ability to tolerate and diagnose faulty processor. The pessimistic diagnosis strategy is a classic strategy based on the PMC model in which isolates all faulty vertices within a set containing at most one fault-free vertex. In this paper, the result that the pessimistic diagnosability $t_p(G)$ equals the extra connectivity $\kappa_1(G)$ of a regular graph G under some conditions are shown. Furthermore, the following new results are gotten: the pessimistic diagnosability $t_p(S_n^2) = 4n - 9$ for split-star networks S_n^2; $t_p(\Gamma_n) = 2n - 4$ for Cayley graphs generated by transposition trees Γ_n; $t_p(\Gamma_n(\Delta)) = 4n - 11$ for Cayley graph generated by the 2-tree $\Gamma_n(\Delta)$; $t_p(BP_n) = 2n - 2$ for the burnt pancake networks BP_n. As corollaries, the known results about the extra connectivity and the pessimistic diagnosability of many famous networks including the alternating group graphs, the alternating group networks, BC networks, the k-ary n-cube networks etc. are obtained directly.

1. Introduction

It is well known that a topological structure of an interconnection network can be modeled by a loopless undirected graph $G = (V, E)$, where vertices in V represent the processors and the edges in E represent the communication links. In this paper, we use graphs and networks interchangeably. The *connectivity* $\kappa(G)$ of a connected graph G is the minimum number of vertices removed to get the graph disconnected or trivial. In a multiprocessor system, some processors may fail, connectivity is used to determine the reliability and fault tolerance of a network. However, a connectivity is not suitable for large-scale processing systems because it is almost impossible for all processors adjacent to, or all links incident to, the same processors to fail simultaneously. To compensate for this shortcoming, it seems reasonable to generalize the notion of classical connectivity by imposing some conditions or restrictions on the components of G when we delete the set of faulty processors. Fábrega and Fiol [17] introduced the *extra connectivity* of interconnection networks as follows.

Definition 1. A vertex set $S \subseteq V(G)$ is called to be an *h-extra vertex cut* if $G - S$ is disconnected and every component of $G - S$ has at least $h + 1$ vertices. The *h-extra connectivity* of G, denoted by $\kappa_h(G)$, is defined as the cardinality of a minimum h-extra vertex cut, if exists.

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It is obvious that $\kappa_0(G) = \kappa(G)$ for any graph G that is not a complete graph. The 1-extra connectivity is usually called extra connectivity. Regarding the computational complexity of the problem [17], there is no known polynomial-time algorithm for finding $\kappa_h(G)$ even for $h = 2$. The problem of determining the extra connectivity of numerous networks has received a great deal of attention in recent years. For a general integer h, Yang and Meng determined the h-extra connectivity of the hypercubes [49] and the folded hypercubes [50], respectively. Chang et al. studied the $\{2, 3\}$-extra connectivity for the hypercube-like networks [3] and the 3-extra connectivity for the folded hypercubes [4]; Hsieh et al. [29] determined the 2-extra connectivity of k-ary n-cubes; Li et al. [35] derived the 3-extra connectivity of the Cayley graphs generated by transposition generating trees; Lin et al. obtained the $\{1, 2, 3\}$-extra connectivity of the split-star networks [37] and the alternating group networks [38], respectively; Guo and Lu [26] studied the h-extra connectivity ($1 \le h \le 3$) of bubble-sort star graphs and Lü [39] obtained the $\{2, 3\}$-extra connectivity of balanced hypercubes etc.

The diagnosis of a system is the process of appraising the faulty processors. A number of models have been proposed for diagnosing faulty processors in a network. Preparata et al. [40] first introduced a graph theoretical model, the so-called *PMC model* (i.e., Preparata, Metze and Chien's model), for system level diagnosis in multiprocessor systems. The pessimistic diagnosis strategy proposed by Kavianpour and Friedman [33] is a classic diagnostic model based on the PMC model. In this strategy, all faulty processors to be isolated within a set having at most one fault-free processor.

Definition 2. A system is *t/t-diagnosable* if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free vertex mistaken as a faulty one. The *pessimistic diagnosability* of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable.

The pessimistic diagnosability of many interconnection networks has been explored. Using the pessimistic strategy, Chwa and Hakimi [12] characterized the diagnosable systems, and Sullivan [42] gave a polynomial time algorithm for determining the diagnosability of a system. Kavianpour and Kim [33] had shown that the hypercubes were $(2n-2)/(2n-2)$-diagnosable. Fan [18] derived the diagnosability of the Möbius cubes using the pessimistic strategy. Wang [47] had shown that the enhanced hypercubes were $2n/2n$-diagnosable. Wang et al. [48] gave the pessimistic diagnosability of the k-ary n-cubes. Tsai in [44] and [45] obtained the pessimistic diagnosability of the alternating group graphs AG_n and the hypercube-like networks (BC networks), respectively. Recently, the pessimistic diagnosability of the (n, k)-arrangement graphs, the (n, k)-star graphs and the balanced hypercubes, the bubble-sort star graphs and augmented k-ary n-cubes were determined in [24] and [25], respectively. For more results related with the diagnosability, you are referred to see [2,20,27,34,36], etc.

Based on the importance of the extra connectivity and the pessimistic diagnosability and motivated by the recent researches on the extra connectivity and pessimistic diagnosability of some graphs, including some famous networks, our object is to propose the relationship between extra connectivity and pessimistic diagnosability of regular graphs with some given conditions. In this paper, the result that the pessimistic diagnosability $t_p(G)$ equals the extra connectivity $\kappa_1(G)$ of a regular graph G under some conditions are shown. Furthermore, the following new results are gotten: the pessimistic diagnosability $t_p(S_n^2) = 4n - 9$ for split-star networks S_n^2; $t_p(\Gamma_n) = 2n - 4$ for Cayley graphs generated by transposition trees Γ_n; $t_p(\Gamma_n(\Delta)) = 4n - 11$ for Cayley graphs generated by the 2-tree $\Gamma_n(\Delta)$; $t_p(BP_n) = 2n - 2$ for the burnt pancake networks BP_n. As corollaries, the known results about the extra connectivity and the pessimistic diagnosability of many famous networks including the alternating group graphs, the alternating group networks, BC networks and the k-ary n-cube networks, etc. are obtained directly.

The remainder of this paper is organized as follows. Section 2 introduces necessary definitions and properties of some graphs. In Section 3, we determines the equal relationship between extra connectivity and pessimistic diagnosability of regular graphs with some given conditions. In Section 4, we concentrates on the applications to some famous networks. The pessimistic diagnosability and the extra connectivity of many famous networks, such as the alternating group graph AG_n, the alternating group network AN_n, the k-ary n-cube networks Q_n^k, the BC networks X_n, the split-star networks S_n^2, the Cayley graphs generated by transposition trees Γ_n, the Cayley graphs generated by 2-trees $\Gamma_n(\Delta)$ and the burnt pancake networks BP_n are obtained directly. Finally, our conclusions are given in Section 5.

2. Preliminaries

In this section, we give some terminologies and notations of combinatorial network theory. For notations not defined here, the reader is referred to [1].

We use a graph, denoted by $G = (V(G), E(G))$, to represent an interconnection network, where $V(G)$ is the vertex set of G; $E(G)$ is the edge set of G. For a vertex $u \in V(G)$, let $N_G(u)$ (or $N(u)$ if there is no ambiguity) denote a set of vertices in G adjacent to u. For a vertex set $U \subseteq V(G)$, let $N_G(U) = \bigcup_{v \in U} N_G(v) - U$ and $G[U]$ be the subgraph of G induced by U. If $|N_G(u)| = k$ for any vertex in G, then G is *k-regular*. For any two vertices u and v in G, let $cn(G; u, v)$ denote the number of vertices who are the neighbors of both u and v, that is, $cn(G; u, v) = |N_G(u) \cap N_G(v)|$. Let $cn(G) = \max\{cn(G; u, v) : u, v \in V(G)\}$, $l(G) = \max\{cn(G; u, v) : (u, v) \in E(G)\}$. Let $|V(G)|$ be the size of vertex set and $|E(G)|$ be the size of edge set. Throughout this paper, all graphs are finite, undirected without loops.