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Abstract

A word w is *rich* if it has $|w|+1$ distinct palindromic factors, including the empty word. A word is *square-free* if it does not have a factor uu, where u is a non-empty word.

Pelantová and Starosta (Discrete Math. 313 (2013)) proved that every infinite rich word contains a square. We will give another proof for that result. Pelantová and Starosta marked with $r(n)$ the length of a longest rich square-free word on an alphabet of size n. The exact value of $r(n)$ was left as an open question. We will give an upper and a lower bound for $r(n)$. The lower bound is conjectured to be exact but it is not explicit.

We will also generalize the notion of repetition threshold for a limited class of infinite words. The repetition thresholds for episturmian and rich words are left as an open question.

Keywords: Combinatorics on words, Palindromes, Rich words, Square-free words, Repetition threshold.
2000 MSC: 68R15

1. Introduction

In recent years, rich words and palindromes have been studied extensively in combinatorics on words. A word is a *palindrome* if it is equal to its reversal. In [DJP], the authors proved that every word w has at most $|w|+1$ distinct palindromic factors, including the empty word. The class of words which achieve this limit was introduced in [BHNR] with the term *full* words. When the authors of [GJWZ] studied these words thoroughly they called them *rich* (in palindromes). Since then, rich words have been studied in various papers, for example in [AFMP], [BDGZ1], [BDGZ2], [DGZ], [RR] and [V].

The *defect* of a finite word w, denoted by $D(w)$, is defined as $D(w) = |w| + 1 - |\mathrm{Pal}(w)|$, where $\mathrm{Pal}(w)$ is the set of palindromic factors in w. The *defect* of an infinite word w is defined as $D(w) = \sup\{D(u) |\ u \text{ is a factor of } w\}$. In other words, the defect is a measure of

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