Accepted Manuscript

Polynomial fixed-parameter algorithms: A case study for longest path on interval graphs

Archontia C. Giannopoulou, George B. Mertzios, Rolf Niedermeier

PII: S0304-3975(17)30422-X
DOI: http://dx.doi.org/10.1016/j.tcs.2017.05.017
Reference: TCS 11182

To appear in: *Theoretical Computer Science*

Received date: 12 April 2017
Revised date: 8 May 2017
Accepted date: 21 May 2017

Polynomial Fixed-Parameter Algorithms: A Case Study for Longest Path on Interval Graphs* †

Archontia C. Giannopoulou‡ § George B. Mertzios¶ Rolf Niedermeier‖

Abstract

We study the design of fixed-parameter algorithms for problems already known to be solvable in polynomial time. The main motivation is to get more efficient algorithms for problems with unattractive polynomial running times. Here, we focus on a fundamental graph problem: LONGEST PATH, that is, given an undirected graph, find a maximum-length path in G. LONGEST PATH is NP-hard in general but known to be solvable in $O(n^4)$ time on n-vertex interval graphs. We show how to solve LONGEST PATH ON INTERVAL GRAPHS, parameterized by vertex deletion number k to proper interval graphs, in $O(k^9 n)$ time. Notably, LONGEST PATH is trivially solvable in linear time on proper interval graphs, and the parameter value k can be approximated up to a factor of 4 in linear time. From a more general perspective, we believe that using parameterized complexity analysis may enable a refined understanding of efficiency aspects for polynomial-time solvable problems similarly to what classical parameterized complexity analysis does for NP-hard problems.

Keywords: polynomial-time algorithm, longest path problem, interval graphs, proper interval vertex deletion set, data reduction, fixed-parameter algorithm.

1 Introduction

Parameterized complexity analysis [20, 22, 24, 43] is a flourishing field dealing with the exact solvability of NP-hard problems. The key idea is to lift classical complexity analysis, rooted in the P versus NP phenomenon, from a one-dimensional to a two- (or even multi-)dimensional perspective, the key concept being "fixed-parameter tractability (FPT)". But why should this natural and successful approach be limited to intractable (i.e., NP-hard) problems? We are convinced that appropriately parameterizing polynomially solvable problems sheds new light on what makes a problem far from being solvable in *linear* time, in the same way as classical FPT algorithms help in illuminating what makes an NP-hard problem far from being solvable in *polynomial* time. In a nutshell, the credo and leitmotif of this paper is that "FPT inside P" is a very interesting, but still too little explored, line of research.

The known results fitting under this leitmotif are somewhat scattered around in the literature and do not systematically refer to or exploit the toolbox of parameterized algorithm design. This should change and "FPT inside P" should be placed on a much wider footing, using parameterized algorithm design techniques such as data reduction and kernelization. As a simple illustrative example, consider the MAXIMUM MATCHING problem. By following a "Buss-like" kernelization (as is standard knowledge in parameterized algorithmics [22, 43]) and then applying a known polynomial-time matching algorithm, it is not difficult to derive an efficient algorithm that, given a graph G with n vertices, computes a matching of size at least k in $O(kn + k^3)$ time. For the sake of completeness we present the details of this algorithm in Section 5.

More formally, and somewhat more generally, we propose the following scenario. Given a problem with instance size n for which there exists an $O(n^c)$-time algorithm, our aim is to identify appropriate parameters k and to derive algorithms with time complexity $f(k) \cdot n^{c'}$ such that $c' < c$, where $f(k)$ depends only on k. First we refine the class FPT by defining, for every polynomially-bounded function $p(n)$, the class FPT($p(n)$) containing the problems solvable in $f(k) \cdot p(n)$ time, where $f(k)$ is an arbitrary (possibly exponential) function of k. It is important to note that, in strong contrast to FPT algorithms for NP-hard problems, here

*A preliminary conference version of this work appears in the *Proceedings of the 10th International Symposium on Parameterized and Exact Computation (IPEC)*, Patras, Greece, September 2015, pages 102–113 [28].

†Partially supported by the EPSRC Grant EP/K022660/1.

‡Institut für Softwaretechnik und Theoretische Informatik, TU Berlin, Germany. Email: archontia.giannopoulou@gmail.com

§The main part of this paper was prepared while the author was affiliated at the School of Engineering and Computing Sciences, Durham University, UK.

¶School of Engineering and Computing Sciences, Durham University, UK. Email: george.mertzios@durham.ac.uk

‖Institut für Softwaretechnik und Theoretische Informatik, TU Berlin, Germany. Email: rolf.niedermeier@tu-berlin.de