Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

On the rectangle escape problem ☆

AmirMahdi Ahmadinejad [a], Sepehr Assadi [b], Ehsan Emamjomeh-Zadeh [c], Sadra Yazdanbod [d], Hamid Zarrabi-Zadeh [e,*]

[a] Management Science and Engineering Department, Stanford University, Stanford, CA 94305, United States
[b] Department of Computer and Information Science, University of Pennsylvania, Philadelphia, PA 19104, United States
[c] Department of Computer Science, University of Southern California, Los Angeles, CA 90089, United States
[d] School of Computer Science, Georgia Institute of Technology, Atlanta, GA 30332-0765, United States
[e] Department of Computer Engineering, Sharif University of Technology, Tehran 14588-89694, Iran

ARTICLE INFO

Article history:
Received 15 April 2016
Received in revised form 20 February 2017
Accepted 31 May 2017
Available online xxxx
Communicated by S.F. Fekete

Keywords:
Rectangle escape
Approximation algorithms
Randomized rounding
NP-completeness
Inapproximability

ABSTRACT

Motivated by the bus escape routing problem in printed circuit boards, we study the following *rectangle escape* problem: given a set S of n axis-aligned rectangles inside an axis-aligned rectangular region R, extend each rectangle in S toward one of the four borders of R so that the maximum density over the region R is minimized. The density of each point $p \in R$ is defined as the number of extended rectangles containing p. We show that the problem is hard to approximate to within a factor better than 3/2 in general. When the optimal density is sufficiently large, we provide a randomized algorithm that achieves an approximation factor of $1+\varepsilon$ with high probability improving over the current best 4-approximation algorithm available for the problem. When the optimal density is one, we develop an exact algorithm that finds an optimal solution efficiently. We also provide approximation algorithms and inapproximability results for a restricted version of the problem where rectangles are allowed to escape toward only a subset of directions.

1. Introduction

In this paper, we revisit the *rectangle escape* problem [5], motivated by a closely related escape routing problem in printed circuit boards. The problem is formally defined as follows:

Problem 1 *(Rectangle escape problem (REP)).* Given an axis-parallel rectangular region R, and a set S of n axis-parallel rectangles inside R, extend each rectangle in S toward one of the four borders of R, so that the maximum density over R is minimized, where the density of a point $p \in R$ is defined as the number of extended rectangles containing p.

To *extend* a rectangle in S, we simply project it onto one of the four borders of R. The bounding box of the rectangle and its projection is called an “extended rectangle”. Note that each extended rectangle contains the area of its original rectangle. An example of the rectangle escape problem is illustrated in Fig. 1. In this example, the maximum density over the region,

☆ A preliminary version of this work was presented at CCCG 2013 [2]. The research was conducted when the first four authors were in the Department of Computer Engineering at Sharif University of Technology.

* Corresponding author.
E-mail addresses: ahmadi@stanford.edu (A. Ahmadinejad), sassadi@cis.upenn.edu (S. Assadi), emamjome@usc.edu (E. Emamjomeh-Zadeh), yazdanbod@gatech.edu (S. Yazdanbod), zarrabi@sharif.edu (H. Zarrabi-Zadeh).

http://dx.doi.org/10.1016/j.tcs.2017.05.040
0304-3975/

Please cite this article in press as: A. Ahmadinejad et al., On the rectangle escape problem, Theoret. Comput. Sci. (2017), http://dx.doi.org/10.1016/j.tcs.2017.05.040

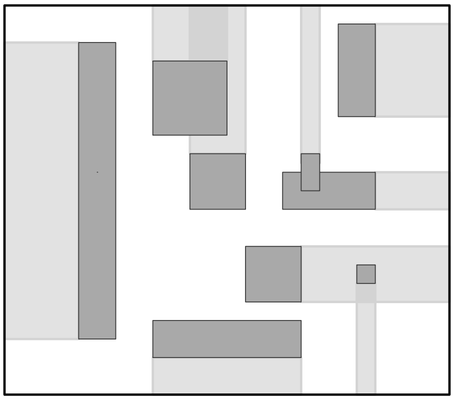

Fig. 1. An instance of the rectangle escape problem. The input rectangles are shown in dark gray, and their projections are shown in light gray.

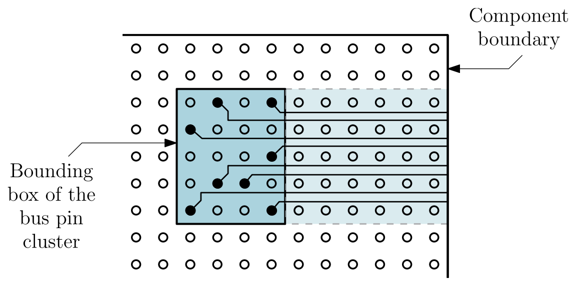

Fig. 2. An example of the bus escape routing problem. The routing is obtained by projecting the bounding box of the bus pin cluster onto the component boundary.

which is equal to the maximum number of extended rectangles that overlap at any point is 2. Not that, by our definition of extended rectangles, the density of the points inside the original rectangles is at least one in any solution.

The study of the rectangle escape problem is motivated by the escape routing problem in printed circuit boards (PCBs). The objective in the *escape routing* problem is to route nets from their pins to the boundary of the enclosing component. The problem has been extensively studied in the literature (see, e.g., [3–7,9–12]). In industrial applications, nets are usually grouped into buses, and the nets from each bus are preferred to be routed together. In this model, the routing of a bus is obtained by projecting the bounding box of the bus onto one of the four sides of the bounding component. An example is illustrated in Fig. 2. The main objective is to find a bus routing in which the maximum number of overlaps at any point is minimized. This is equivalent to the rectangle escape problem, as defined in Problem 1.

The rectangle escape problem is known to be NP-hard [5]. The *decision version* of the problem, which we call k-REP, is defined as follows:

Problem 2 *(k-REP).* Given an instance of the rectangle escape problem and an integer $k \geq 1$, determine whether any routing is possible with a density of at most k.

It is known that the k-REP problem is NP-complete, even for $k = 3$ [5]. The best current approximation algorithm for the optimization version of the problem is due to Ma and Wong [5] that achieves an approximation factor of 4, using a deterministic linear programming (LP) rounding technique.

For a special case when the optimal density is 1 (i.e., when all chips can be routed with no conflict), the problem can be solved exactly using polynomial-time algorithms available for the related *maximum disjoint subset* problem [1,3].

Our results In this paper, we obtain some results on the rectangle escape problem, a summary of which is listed below.

- We show that the k-REP problem is NP-complete for any $k \geq 2$. Given that the problem is polynomially solvable for $k = 1$, this fully settles the complexity of the problem for all values of k. An important implication of this result is that the rectangle escape problem is hard to approximate to within any factor better than 3/2, unless P = NP.
- Despite the fact that the problem is hard to approximate to within a constant factor when the optimal density is low, we present a randomized algorithm that achieves an approximation factor of $1 + \varepsilon$ with high probability, when the optimal density is at least $c_\varepsilon \log n$, for some constant c_ε. This improves, for instances with high density, upon the current best