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Computing conforming partitions of orthogonal polygons with minimum stabbing number ☆

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Let P be an orthogonal polygon with n vertices. A partition of P into rectangles is called *conforming* if it results from cutting P along a set of interior-disjoint line segments, each having both endpoints on the boundary of P. The stabbing number of a partition of P into rectangles is the maximum number of rectangles stabbed by any orthogonal line segment inside P. In this paper, we consider the problem of finding a conforming partition of P with minimum stabbing number. We first give an $O(n \log n)$-time algorithm to solve the problem when P is a histogram. For an arbitrary orthogonal polygon (even with holes), we give an integer programming formulation of the problem and show that a simple rounding results in a 2-approximation algorithm for the problem. Finally, we show that the problem is NP-hard if P is allowed to have holes.

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1. Introduction

The problem of partitioning a polygonal shape into simpler components is a well-studied problem in computational geometry, with many applications in other areas of research including VLSI layout design [3,4], chip manufacturing [5], geoinformatics [6], image processing [7], and pattern recognition [8,9]. Previous related research in this area was focused on "convexity"; that is, partitioning polygons into convex regions so as to minimize the number of convex components [10–14]. Another optimality criterion studied in the literature is to minimize the total length of partition segments [15–19]. Another line of research focused on restricting the shape of the input polygon, among which orthogonal polygons were frequently studied as natural polygonal shapes. For instance, in their seminal paper, Lingas et al. [15] showed that minimizing the total length of partition segments on a simple orthogonal polygon is polynomial-time solvable, while the problem becomes NP-hard if the polygon is allowed to have holes [15]. Moreover, Gonzalez and Zheng [20,21] studied the approximability of the same problem exclusively on orthogonal polygons with additional constraint that the partition segments must pass through a given set of points in the polygon (see also [22]).

Preliminaries and definitions A polygon P is orthogonal if all of its edges are either vertical or horizontal. A *rectangular partition* of an orthogonal polygon P is a set of interior-disjoint rectangles whose union is P. Let R be a rectangular partition

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of an orthogonal polygon P. Given a line segment ℓ inside P, we say that ℓ *stabs* a rectangle of R if ℓ passes through the interior of the rectangle. The *orthogonal stabbing number* of R is the maximum number of rectangles of R stabbed by any orthogonal line segment inside P. We define the *vertical* (resp., *horizontal*) stabbing number of R as the maximum number of rectangles stabbed by any vertical (resp., horizontal) line segment inside P. For the rest of this paper, "stabbing" is assumed to be orthogonal stabbing, unless noted otherwise. A rectangular partition of P is called *conforming* if it corresponds to the faces of the arrangement of a set of line segments in P, such that each line segment has both endpoints on the boundary of P, and no two line segments intersect, except possibly at their endpoints on the boundary of P. In this paper, we study the *Optimal Conforming Partition* problem: given an orthogonal polygon, the objective is to compute a conforming partition of the polygon whose stabbing number is minimum over all such partitions of the polygon.

Let R be a conforming partition of P. We refer to an edge of a rectangle of R that is not a subset of an edge of P a *partition edge*. That is, the partition edges of R correspond to the "cuts" that divide P into rectangles. A vertex u of P is a *reflex* vertex if the angle at u interior to P is $3\pi/2$. We denote the set of reflex vertices of P by $\texttt{reflexV}(P)$. For each reflex vertex $u \in \texttt{reflexV}(P)$, we denote the maximal horizontal (resp., vertical) line segment contained in the interior of P with one endpoint at u by H_u (resp., V_u) and refer to it as the *horizontal line segment* (resp., *vertical line segment*) of u. Observe that for every reflex vertex u of P, at least one of H_u and V_u must be present in R. The following observation allows us to consider only a discrete subset of the set of all possible rectangular partitions of P to find an optimal conforming partition:

Observation 1. Any orthogonal polygon P has an optimal conforming partition in which every partition edge is either H_u or V_u for some $u \in \texttt{reflexV}(P)$.

Related work It is shown by de Berg and van Kreveld [23] that every n-vertex orthogonal polygon has a rectangular (not necessarily conforming) partition with stabbing number $O(\log n)$. They show that this bound is asymptotically tight, as the stabbing number of any rectangular partition of a staircase polygon with n vertices is $\Omega(\log n)$. Independently, de Berg and van Kreveld [23] and Hershberger and Suri [24] gave polynomial-time algorithms that compute partitions with stabbing number $O(\log n)$. Recently, Abam et al. [25] considered the problem of computing an optimal rectangular partition of a simple orthogonal polygon; that is, a rectangular partition (not restricted to being conforming) whose stabbing number is minimum over all such partitions of the polygon. By finding an optimal partition for histograms in $O(n^7 \log n \log\log n)$ time, they obtained a 3-approximation algorithm for this problem. The complexity of finding an optimal partition for an arbitrary orthogonal polygon remains open.

Minimizing the stabbing number of partitions of other inputs are also studied. For instance, de Berg et al. [26] studied the problem of partitioning a given set of n points in $\mathbb{R}^d$ into sets of cardinality between $n/2r$ and $2n/r$ for a given r, where each set is represented by its bounding box, such that the stabbing number is minimized. Here, the stabbing number is defined as the maximum number of bounding boxes intersected by any axis-parallel hyperplane. They showed that the problem is NP-hard in $\mathbb{R}^2$. They also gave an exact $O(n^{4dr+3/2} \log^2 n)$-time algorithm in $\mathbb{R}^d$ as well as an $O(n^{3/2} \log^2 n)$-time 2-approximation algorithm in $\mathbb{R}^2$ when r is constant. Fekete et al. [27] proved that the problem of finding a perfect matching with minimum stabbing number for a given point set is NP-hard, where the stabbing number of a matching is the maximum number of edges of the matching intersected by any axis-parallel line. They also showed that the problems of finding a spanning tree or a triangulation of a given point set with minimum stabbing number are NP-hard.

Our results This paper examines the problem of finding an optimal conforming partition of an orthogonal polygon. First, we give an $O(n \log n)$-time algorithm for computing an optimal partition when the input polygon is a histogram with n vertices (Section 2). Next, we give a polynomial-time 2-approximation algorithm for the problem on arbitrary orthogonal polygons, even with holes (Section 3). Finally, we show the NP-hardness of the optimal conforming partition problem on orthogonal polygons with holes in Section 4. To the authors' knowledge, this is the first complexity result related to determining the minimum stabbing number of a rectangular partition of an orthogonal polygon. We conclude the paper with a discussion on open problems in Section 5.

2. Histograms

In this section, we give an $O(n \log n)$-time algorithm for the optimal conforming problem on a histogram with n vertices. A *histogram* (polygon) H is a simple orthogonal polygon that has one edge e that can see every point in P. More formally, H is a vertical (resp., horizontal) histogram if it is monotone with respect to some horizontal (resp., vertical) edge e on the boundary of P [28,29]; i.e., e spans the width (resp., height) of P. We call e the *base* of H. For the rest of this section, we assume that H is a vertical histogram with n vertices.

We note that Abam et al. [25] gave a polynomial-time algorithm for computing an optimal rectangular partition of a histogram; their algorithm may not necessarily produce a conforming partition. Fig. 1 shows a histogram whose optimal rectangular partition has stabbing number 2, while any conforming partition of this histogram has stabbing number at least 3.

Let H^- denote the set of horizontal edges of H. Recall by Observation 1 that every conforming partition of H must include at least one of the edges H_u or V_u for every reflex vertex u in H. The algorithm begins with an initial partition of H, consisting of all horizontal partition edges, that will be modified to produce an optimal conforming partition of H

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