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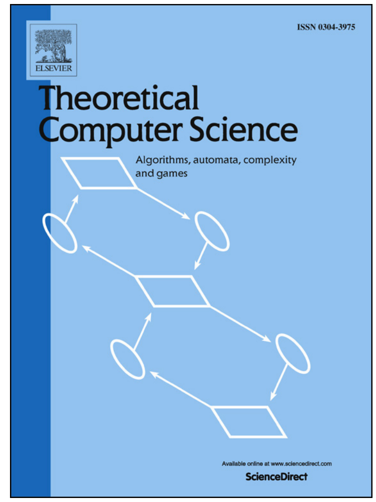

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Neighbor sum distinguishing total coloring of planar graphs without 5-cycles

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Abstract Let G be a graph, a proper total coloring $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ is called *neighbor sum distinguishing* if $f(u) \neq f(v)$ for each edge $uv \in E(G)$, where $f(v) = \sum_{uv \in E(G)} \phi(uv) + \phi(v)$, $v \in V(G)$. We use $\chi''_{\Sigma}(G)$ to denote the smallest number k in such a coloring of G. Pilśniak and Woźniak have already conjectured that $\chi''_{\Sigma}(G) \leq \Delta(G) + 3$ for any simple graph with maximum degree $\Delta(G)$. In this paper, we prove that for any planar graph G without 5-cycles, $\chi''_{\Sigma}(G) \leq \max\{\Delta(G) + 3, 10\}$.

Keywords Neighbor sum distinguishing total coloring; Combinatorial Nullstellensatz; Planar graph

1 Introduction

For the terminology and notation not defined in this paper, we follow [2]. All graphs considered in this paper are simple, finite and undirected. Let $G = (V(G), E(G))$ be a graph with maximum degree $\Delta(G)$. Let $N(v)$ denote the neighbor set of a vertex v in $V(G)$. A vertex v of degree t is called a *t-vertex*. A *t^--vertex* (or *t^+-vertex*) is a vertex of degree at most t (or at least t). An edge e of graph G is a *cut-edge* if $G - e$ contains more connected components than G. Let $G = (V(G), E(G), F(G))$ be a plane graph. The *degree* of a face f in G, denoted by $d_G(f)$ (or $d(f)$), is the number of edges incident with it, where each cut-edge is counted twice. A face f of degree l is called an *l-face*. An l-face $v_1 v_2 \cdots v_l$ is a $(b_1, b_2, \ldots, b_l)$-*face*, if v_i is a b_i-vertex, for $i = 1, 2, \ldots, l$.

The famous four color problem has stated as an initial point of the development of graph coloring. Graph coloring has a wide range of application in many fields, such as scheduling problem, storage problem, electric-network problem and so on. In graph coloring, vertex coloring and total coloring are two fundamental colorings. A *proper k-vertex coloring* of graph G is a coloring of $V(G)$ using k colors such that no two adjacent vertices receive the same color. A *proper total k-coloring* of G is a coloring of $V(G) \cup E(G)$ using k colors in the sense that no two adjacent or incident elements receive the same color. In this paper, all colorings are proper colorings.

Total coloring is the cornerstone of many special colorings. In this paper, we focus on another interesting coloring in the environment of total coloring. Given a graph G and a total k-coloring ϕ: $V(G) \cup E(G) \to \{1, 2, \ldots, k\}$. Let $f(v) = \sum_{uv \in E(G)} \phi(uv) + \phi(v)$, $v \in V(G)$. If $f(u) \neq f(v)$ for each edge $uv \in E(G)$, the coloring ϕ is *k-neighbor sum distinguishing total coloring*, or *k-tnsd-coloring* for simplicity. The smallest number k in such a coloring of graph G is *neighbor sum distinguishing total chromatic number*, denoted by $\chi''_{\Sigma}(G)$. Clearly, a tnsd-coloring of a graph G is a strengthening total coloring, which induces a vertex coloring of G. For k-tnsd-coloring, Pilśniak and Woźniak put forward the following conjecture.

Conjecture 1.1 [8] *For any graph G, $\chi''_{\Sigma}(G) \leq \Delta(G) + 3$.*

Pilśniak and Woźniak also verified that Conjecture 1.1 holds for some special graphs. For planar graphs, Li et al. [6] showed this conjecture for $\Delta(G) \geq 13$, and subsequently the result was improved by Qu et al. [9]. Recently, Song et al. [10] proved that $\chi''_{\Sigma}(G) \leq \max\{\Delta(G) + 2, 14\}$ for planar graphs. Wang et al. [13] verified this conjecture for any triangle free planar graph G with $\Delta(G) \geq 7$. Wang et al. [11] showed that

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