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Minimal reaction systems: Duration and blips

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ABSTRACT

We investigate *reaction systems* introduced in [5], in particular, the subclass of *minimal* reaction systems added with a feature of *duration*. It turns out that the model is computationally strong. Moreover, in some cases the lengths of the resulting sequences and cycles can be found out directly by arithmetical properties of the duration values.

1. Introduction

This paper falls in the line of research, very successful recently, dealing with *reaction systems*, a model for computing introduced by Ehrenfeucht and Rozenberg, [5]. From a purely mathematical point of view, the model offers a new approach in the study of functions from the set of subsets of a finite set S into itself. A *reaction* is simply a triple (R, I, P) *(reactants, inhibitors, products)* of nonempty subsets of S such that R and I do not intersect.

Many variants, extensions and modifications of reaction systems have been introduced but we investigate the very basic model added with a feature of *duration*: each element has the possibility of staying around for some moments. Such a feature was introduced in [2] and investigated further in [15].

We are concerned with mathematical properties of the model of reaction systems rather than with eventual applications. Related work is contained in [4,7,10,9,11–15]. The model, provided with many variants and additions, turned out to be suitable in different setups. The paper [1] constitutes a survey. A related very general approach is presented in [6].

A widely investigated subclass of reaction systems has been the *minimal* ones, where the sets R and I consist of one element each. The reference [4] characterizes the class of functions definable by minimal reaction systems. It turns out that the class is very limited in comparison with the class of functions definable by arbitrary reaction systems. However, this defect can be compensated using some simple variations of reaction systems. One approach was presented in [14].

Another possibility is investigated in this paper. We consider *full* minimal reaction systems with duration, that is, there is a reaction in the system for every pair of reactant and inhibitor. It turns out that such a model is computationally strong. Moreover, the lengths of the resulting sequences and cycles can be found out, using the Chinese Remainder Theorem, by arithmetical properties of the duration values.

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2. Definitions and auxiliary results

We will now present formal definitions for the basic notions discussed in the paper. We begin with the central notion of a reaction system, as well as related operational notions. Everything happens within the framework of a finite set S, the *background set*. Essentially, we are dealing with functions from the set of subsets of S into itself. Throughout the paper, S refers to the background set.

Definition 1. A *reaction* over the *finite background set* S is a triple

$$\rho = (R, I, P),$$

where R, I and P are nonempty subsets of S such that R and I do not intersect. The three sets are referred as *reactants, inhibitors* and *products*, respectively. A *reaction system* $\mathcal{A}_S$ over the background set S is a finite nonempty set

$$\mathcal{A}_S = \{\rho_i |\ 1 \leq i \leq k\},\ k \geq 1,$$

of reactions over S.

Note that P may or may not contain elements of $R \cup I$. The *cardinality* of a finite set X is denoted by $\sharp X$. The *empty set* is denoted by $\emptyset$.

We will omit the index S from $\mathcal{A}_S$ whenever S is understood. We now come to the definitions dealing with functions and sequences.

Definition 2. Consider a reaction $\rho = (R, I, P)$ over S and a subset T of S. The set T is *enabled* (with respect to ρ), in symbols $en_\rho(T)$, if $R \subseteq T$ and $I \cap T = \emptyset$. If T is (resp. is not) enabled, then we define the *result* by

$$res_\rho(T) = P \text{ (resp. } = \emptyset).$$

For a reaction system $\mathcal{A} = \{\rho_j |\ 1 \leq j \leq k\}$, we define the *result* by

$$res_{\mathcal{A}}(T) = \bigcup_{j=1}^{k} res_{\rho_j}(T).$$

Definition 2 exhibits an important feature of reaction systems. Whenever an element is in a set, it is considered to be present for all reactions simultaneously. Thus, an element is not "consumed" in the application of the reaction but is also available for other reactions. In this sense no "conflicts" arise. This feature makes reaction systems different from many other models for computation.

Sequences generated by reaction systems can be viewed as iterations of the operation $res_{\mathcal{A}}$. If $res_{\mathcal{A}}(Y) = Y'$, we use the notation

$$Y \Rightarrow_{\mathcal{A}} Y',$$

or simply $Y \Rightarrow Y'$. If

$$res_{\mathcal{A}}(X_i) = X_{i+1},\ 0 \leq i \leq m-1,$$

we write briefly

$$X_0 \Rightarrow X_1 \Rightarrow \ldots \Rightarrow X_m$$

and call $X_0, X_1, \ldots, X_m$ *states* of a *sequence* of *length* $m+1$ generated (or defined) by the reaction system $\mathcal{A}$. The reflexive and transitive closure of the relation $\Rightarrow$ is denoted by $\Rightarrow^*$.

It is important to notice that, whenever in a sequence $X_i = X_j$, $i \neq j$, then also $X_{i+1} = X_{j+1}$. As we will see, reaction systems with duration do not satisfy this condition.

Since there are only $2^{\sharp S}$ subsets of S, one of the following two situations always occurs for a sequence

$$X_0 \Rightarrow X_1 \Rightarrow \ldots \Rightarrow X_{m-1},$$

for large enough m.

1. $res_{\mathcal{A}}(X_{m-1}) = X_{m_1}$, for some $m_1 \leq m-1$. If m_1 is the largest number satisfying this inequality, we say that the sequence has (or ends with) a *cycle* of length $m - m_1$.

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