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Decision making with an interval-valued fuzzy preference relation and admissible orders

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ABSTRACT

In this paper we analyze under which conditions we must use interval-valued fuzzy relations in decision making problems. We propose an algorithm to select the best alternative from a set of solutions which have been calculated with the nondominance algorithm using intervals and different linear orders among them. Based on the study made by Orlovsky in his work about nondominance, we study a characterization of weak transitive and 0.5-transitive interval-valued fuzzy relations, as well as the conditions under which transitivity is preserved by some operators on those relations. Next, we study the case of interval-valued reciprocal relations. In particular, we describe the preservation of reciprocity by different operators and analyze the transitivity properties for these interval-valued reciprocal relations. Finally, we propose to use, in the nondominance algorithm, linear interval orders generated by means of operators which preserve transitivity.

1. Introduction

A decision making problem with an expert can be summarized as follows: we have a set of p alternatives

$X = \{x_1, \ldots, x_p\}$ with $p \geq 2$

and the expert provides his/her preferences on the former set of alternatives. We must find an alternative solution for the considered expert.

Depending on the nature of alternatives and of the knowledge of the expert about them, preferences can be expressed in different ways. In this paper we consider that this expression is done using fuzzy sets (see [1,17,32,33,41,43,49]).

We assume that the expert provides his/her preferences on the set of alternatives using a fuzzy binary relation R on X defined as a fuzzy subset of $X \times X$; that is, $R : X \times X \to [0, 1]$. The value $R(x_i, x_j) = R_{ij}$ denotes the degree to which elements x_i and x_j are related in the relation R for all $x_i, x_j \in X$ (see [28]). Particularly, in preference analysis, R_{ij}, denotes the degree to which alternative x_i is preferred to alternative x_j. We say that a fuzzy preference relation R satisfies the property of *reciprocity* if $R_{ij} + R_{ji} = 1$ for all $i, j \in \{1, \ldots, n\}$. In reciprocal preference relations it is usual not to define the elements in the diagonal or to take the value 0.5 [37] (see also [39,40]).

$$R = \begin{pmatrix} 0.5 & R_{12} & \cdots & R_{1p} \\ R_{21} & 0.5 & \cdots & R_{2p} \\ \cdots & \cdots & 0.5 & \cdots \\ R_{p1} & \cdots & \cdots & 0.5 \end{pmatrix} \tag{1}$$

There exist different methods to find an alternative as solution from R. One of the most widely used is the weighted vote (see [35,36]): Given a preference relation as that of Eq. (1) the weighted vote strategy consists in taking as preferred alternative the solution of:

$$arg \max_{i=1,\ldots,p} \sum_{1 \leq j \neq i \leq p} R_{ij} \tag{2}$$

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However, in some situations this method does not allow us to choose an alternative as solution in a unique way. For instance, consider the following preference relation for $p = 4$:

$$R = (R_{ij})_{4\times 4} = \begin{pmatrix} 0.5 & 0.3149 & 0.1605 & 0.3640 \\ 0.6851 & 0.5 & 0.0407 & 0.0624 \\ 0.8395 & 0.9593 & 0.5 & 0.3874 \\ 0.6360 & 0.9376 & 0.6126 & 0.5 \end{pmatrix} \quad (3)$$

Adding the elements of each of the rows of R we have: (1.3394, 1.2882, 2.6862, 2.6862). So, with this method, we have two possible solutions, x_3 and x_4, and we do not know which one we should take if we must pick up only one.

When this happens, either with the voting method or with some other strategy, sometimes it is advisable to apply a different algorithm. Historically, one of the most widely used methods is the one given by Orlovsky in 1978 and called nondominance method [41]. This method extracts as the solution the least dominated alternative(s) of the fuzzy decision making problem starting from a fuzzy preference relation. Specifically, the maximal nondominated elements of a normalized fuzzy preference relation R are calculated by means of the following operations:

1 Compute the fuzzy strict preference relation:

$$R^s_{ij} = \begin{cases} R_{ij} - R_{ji} & \text{if } R_{ij} > R_{ji} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

2 Compute the nondominance degree of each alternative: $ND_i = 1 - \bigvee_j R^s_{ji}$;

3 Select as alternative: $Alternative(x_p) = arg \max_{i=1,\ldots n} \{ND_i\}$.

Nevertheless, with this method, the following may happen:

(a) there may exist two or more alternatives with the same nondominance degree, i.e. a total ordering of the set of alternatives is not guaranteed or

(b) all the alternatives may have a similar nondominance degree and we select an alternative as solution but we are not sure about the alternative elected. Suppose after applying the first step of the Orlovsky's algorithm we obtain the following strict fuzzy preference relation:

$$R^s = \begin{pmatrix} 0.5 & 0 & 0 & 0.83 \\ 0.84 & 0.5 & 0.84 & 0 \\ 0.85 & 0 & 0.5 & 0 \\ 0 & 0.86 & 0.81 & 0.5 \end{pmatrix}$$

If we compute the second step we obtain: $ND_1 = 1 - 0.85 = 0.15$, $ND_2 = 1 - 0.86 = 0.14$, $ND_3 = 1 - 0.84 = 0.16$ and $ND_4 = 1 - 0.83 = 0.17$. Therefore x_4 will be elected as solution but we can conclude that nondominance over alternatives are quite similar.

All these considerations lead us to propose the use of interval-valued fuzzy relations (IVFRs) when other widely used decision making methods do not allow us to choose a single alternative as solution and we need to select one and only one. Moreover, the use of intervals [12,16] enables us to improve the representation of the preferences, since they allow us to incorporate, for instance, the uncertainty of the original preference values given by the experts by means of the length of the intervals.

Even more, there are not deep theoretical studies of the use of the nondominance degrees in an interval-valued setting with more than one linear order as the work by Orlovsky. For this reason, the two first goals of this paper are:

- To develop a theoretical study of the properties of IVFRs similar to the one done by Orlovsky for his fuzzy nondominance method [41]. This objective makes us study the properties of transitivity and 0.5-transitivity of IVFRs, as well as the dependence between Interval-Valued Fuzzy Reciprocal Relations (IVFRRs) and transitivity (for different types of transitivity properties).
- To introduce the nondominance method for IVFRs.

In the nondominance method, in order to choose the best alternative, we must order the values and take as solution the alternative linked to the highest value. Clearly, when we are working with numbers, a linear order is at our disposal, and we are always able to say which number is the biggest one. So, in order to work with intervals, the first thing we must do is to pick up a linear order between intervals [14,15], since it may happen that two intervals are not comparable by means of the usual (partial) order between intervals [4].

There exist different ways to build linear orders between intervals which extend the usual partial order and which make use of aggregation functions. In this paper, we are going to use the method in [14] using the operators $F_{\alpha,\beta}$ [8,10]. For this reason, we also pose the following objective.

- To study under which conditions the operators $F_{\alpha,\beta}$ and other operators defined in [2] preserve the usually demanded properties of IVFRs (transitivity and the reciprocity property).

It is clear that the use of the nondominance method does not guarantee that we can select a unique solution alternative. However, the use of IVFRs allows us to apply the nondominance method several times with a different linear order each of them. In this way, we will take as solution the alternative which appears most often in the first place in the different solutions that we get.

This work is composed of three different parts. The first one is devoted to Interval-Valued Fuzzy Relation (IVFRs), while the second one tackles the case of IVFRRs. In Section 2 we review some relevant concepts about IVFRs. Section 3 is devoted to the study of transitivity properties of IVFRs and the action of some selected operators related to IVFRs. We introduce here the concept of 0.5-transitivity, characterize weak transitivity and 0.5-transitivity. Moreover, we consider an equivalence relation for IVFRs and study its connection with the transitivity property. We also put an interpretation of the considered concepts. In the second part, which starts in Section 4 we focus our attention to the case of the IVFRRs to consider, among other concepts, transitivity of IVFRRs (we propose some suitable assumptions for IVFRRs to fulfill the weak transitivity) and study the preservation of reciprocity and transitivity under the application of some relevant operators. In the third part of the paper, Section 5, we discuss the nondominance algorithm for IVFRs. We also show an example. In Section 6, we propose a method to take the solution alternative using the nondominance algorithm with different linear interval orders. To finish, we present some conclusions and references.

2. Preliminaries

In this section we recall the relevant concepts about IVFRs. We also present some results about interval-valued preference relations.