

# Accepted Manuscript

Newton's method in practice: finding all roots of polynomials of degree one million efficiently

Dierk Schleicher, Robin Stoll

PII: S0304-3975(17)30247-5  
DOI: <http://dx.doi.org/10.1016/j.tcs.2017.03.025>  
Reference: TCS 11125

To appear in: *Theoretical Computer Science*



Please cite this article in press as: D. Schleicher, R. Stoll, Newton's method in practice: finding all roots of polynomials of degree one million efficiently, *Theoret. Comput. Sci.* (2017), <http://dx.doi.org/10.1016/j.tcs.2017.03.025>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# NEWTON'S METHOD IN PRACTICE: FINDING ALL ROOTS OF POLYNOMIALS OF DEGREE ONE MILLION EFFICIENTLY

DIERK SCHLEICHER AND ROBIN STOLL

ABSTRACT. We use Newton's method to find all roots of several polynomials in one complex variable of degree up to and exceeding one million and show that the method, applied to appropriately chosen starting points, can be turned into an algorithm that can be applied routinely to find all roots without deflation and with the inherent numerical stability of Newton's method.

We specify an algorithm that provably terminates and finds all roots of any polynomial of arbitrary degree, provided all roots are distinct and exact computation is available. It is known that Newton's method is inherently stable, so computing errors do not accumulate; we provide an exact bound on how much numerical precision is sufficient.

## 1. INTRODUCTION

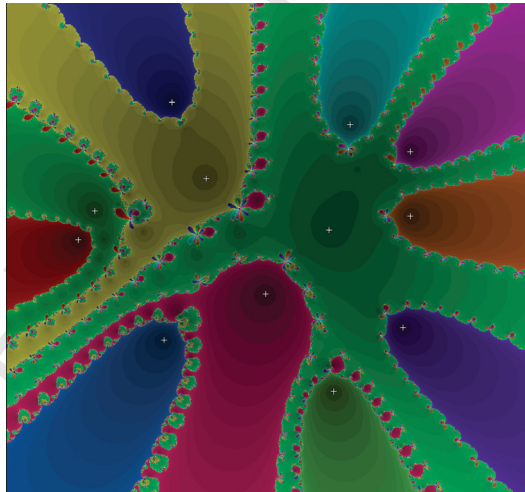


FIGURE 1. The dynamics of Newton's method for a polynomial of degree 12. Different colors indicate starting points that converge to different roots, and different shades of color indicate the speed of convergence to that root.

Finding roots of equations, especially polynomial equations, is one of the oldest tasks in mathematics; solving any equation  $f(x) = g(x)$  means finding roots of  $(f - g)(x)$ . This task is of fundamental importance in modern computer algebra systems, as well as for geometric modelling. Newton's method, as the name indicates, is one of the oldest methods for approximating roots of smooth maps, and in many cases

Download English Version:

<https://daneshyari.com/en/article/4952085>

Download Persian Version:

<https://daneshyari.com/article/4952085>

[Daneshyari.com](https://daneshyari.com)