



Consistency of the fused intuitionistic fuzzy preference relation in group intuitionistic fuzzy analytic hierarchy process



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ARTICLE INFO

Article history:

Received 18 September 2014

Received in revised form 4 April 2015

Accepted 5 April 2015

Available online 16 April 2015

Keywords:

Atanassov's intuitionistic fuzzy set (A-IFS)

Intuitionistic fuzzy preference relation

Multiplicative consistency

Group decision making

Group intuitionistic fuzzy analytic

hierarchy process (GIFAHP)

ABSTRACT

Intuitionistic fuzzy preference relations (IFPRs), which are based on Atanassov's intuitionistic fuzzy sets (A-IFS), have turned out to be a useful structure in expressing the experts' uncertain judgments, and the intuitionistic fuzzy analytic hierarchy process (IFAHP) is a method for solving multiple criteria decision making problems. To provide a theoretical support for group decision making with IFAHP, this paper presents some straightforward and useful results regarding to the aggregation of IFPRs. Firstly, a new type of aggregation operator, namely, simple intuitionistic fuzzy weighted geometric (SIFWG) operator, is developed to synthesize individual IFPRs. It is well known that for traditional comparison matrices, if all individual comparison matrices are of acceptable consistency, then their weighted geometric mean complex judgment matrix is of acceptable consistency. In this paper, we prove that this property holds for IFPRs as well if we use the SIFWG operator to synthesize the individual IFPRs. A numerical example is given to verify the theorems. It is pointed out that the well-known simple intuitionistic fuzzy weighted averaging (SIFWA) operator, the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy weighted geometric (IFWG) operator and the symmetric intuitionistic fuzzy weighted geometric (SYIFWG) operator do not have this property. Finally, the group IFAHP (GIFAHP) procedure is developed to aid group decision making process with IFPRs.

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1. Introduction

In many practical decision making situations, such as choosing a car to buy or selecting a person for higher managing position, the decision makers or experts may prefer to express their preference information by comparing each pair of objects and construct a preference relation which stores their preference information over a set of alternatives or criteria in a matrix. There are many different types of preference relations, such as multiplicative preference relations (MPRs) [1], fuzzy preference relations (FPRs) [2], and intuitionistic fuzzy preference relations (IFPRs) [3] which are based on Atanassov's intuitionistic fuzzy sets (A-IFSs). Xu [4] made a survey on different kinds of preference relations and discussed their properties. The goal of establishing a preference relation is to derive the priority weights of objects from the preference relation and then rank the objects according to the priority weights [5]. However, in many cases, the preference relation may not be consistent. Consistency of a preference relation requires that the expert's judgments yield no contradiction. Due to the fact that the lack of consistency for a preference relation can lead to inconsistent or incorrect conclusion, the consistency of a preference relation turns out to be a very important research topic, which has been attracting more and more scholars' attention.

The earliest work on consistency was done by Saaty [1], who proposed a consistency ratio from a MPR and suggested that a MPR is of acceptable consistency if its consistency ratio is less than 0.1. He also presented that it is difficult to obtain such MPR, especially when the MPR has a high order. For the inconsistent MPR, two ways can be used to deal with such kind of MPR: one is to return such an inconsistent MPR to the decision maker to reconsider constructing new MPR until the acceptable consistency is reached [6]; the other way is to improve the inconsistent MPR automatically by some iterative algorithms [7,8]. The first method is accurate and reliable but wastes a lot of time, and in some settings, if the decision makers do not want to interact with the experts, or if they cannot find the initial experts to re-evaluate

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and alter their preferences, or if consistency must be urgently obtained, the feedback mechanism is out of use [5]. The iterative algorithms to improve the consistency of a MPR involve two sorts: modifying a single element [7] and modifying all elements [8].

Transitivity is the most important concept for consistency issue. As for FPR, Tanino [9,10] introduced the weak transitivity, the max–min transitivity, the max–max transitivity, the restricted max–min transitivity, the restricted max–max transitivity, the additive transitivity and the multiplicative transitivity for a FPR. The weak transitivity is the minimum requirement condition to find out whether a FPR is consistent or not [11]. The max–max transitivity, the max–min transitivity, the restricted max–min transitivity and the restricted max–max transitivity do not imply reciprocity [12]. Both the additive transitivity and the multiplicative transitivity imply reciprocity, and thus have been used widely in practical applications. Although the additive consistency of a FPR is equivalent to Saaty's consistency property of a MPR [11,13], it is in conflict with the [0,1] scale used for providing the preference values [13]. Therefore, it is an inappropriate property to model the consistency of FPR [14]. Xia and Xu [15] proposed an iterative algorithm based on the multiplicative consistency of a FPR to improve the consistency of a FPR until it is of acceptable consistency. They also proved that if all individual FPRs are multiplicative consistent, then their fused FPR is multiplicative consistent.

With respect to IFPR, different kinds of consistency were proposed. Via the transformation between the IFPR and its corresponding interval-valued fuzzy preference relation (IVFPR), Xu [16] introduced a definition of additive consistent IFPR, which is based on the additive consistent IVFPR. Gong et al. [17] proposed another form of definition for additive consistent IFPR. Later, Wang [18] directly employed the membership and non-membership degrees to define the additive consistent IFPR. Based on the corresponding converted interval-valued membership degrees, Gong et al. [19] introduced the definition of multiplicative consistent IFPR. Xu et al. [20] also proposed another definition of multiplicative consistent IFPR based on the membership and non-membership degrees. Due to the fact that Gong et al.'s [19] definition of multiplicative consistent IFPR is not based on the Atanassov's intuitionistic fuzzy judgments directly, and Xu et al.'s [20] definition is not reasonable in some cases, Liao and Xu [21] introduced a novel form of definition of multiplicative consistent IFPR based on the membership and non-membership degrees of the decision maker's Atanassov's intuitionistic fuzzy judgments.

It should be noted that the elements in an IFPR are intuitionistic fuzzy values (IFVs) [22] which are composed of a membership degree, a non-membership degree and a hesitancy degree. Due to the powerfulness of A-IFVs in describing fuzziness and uncertainty [23–25], the IFPR is more useful than the MPR and the FPR in expressing comprehensive preference information. Nowadays, more and more scholars and practitioners applied the IFPR into practical decision making problems. Xu [3] developed an approach to group decision making based on IFPRs and then used it to assess the agroecological regions in Hubei Province, China. In order to handle complex decision making problems, Xu and Liao [26] extended the classical AHP method to the IFAHP and then employed it to global supplier development problem which includes both qualitative and quantitative factors. Liao and Xu [21] investigated the intuitionistic fuzzy priority derivation methods for an IFPR and then applied the methods in selecting the flexible manufacturing systems for a company. Later, they [27] also proposed some fractional models to determine the intuitionistic fuzzy priorities from the IFPRs in group decision making, and implemented the methods in evaluating the candidate exchange doctoral students from all over the world. Recently, Xu and Liao [28] made a state of the art survey of approaches to decision making with IFPR. In this paper, we focus our attention on group decision making with IFPRs.

Group decision making, which involves diverse decision makers' or experts' opinions, takes place commonly in our daily life. In group decision making with any types of preference relations, the most important issue is how to aggregate all the experts' preference information into reliable collective preference information. Forman and Peniwati [29] described two basic aggregation methods:

- Aggregating individual priorities. The aggregation of individual priorities is suitable when the group acts as separate individuals. In such a case, the weighted arithmetic mean (WAM) method is usually used.
- Aggregating individual judgments. The aggregation of individual judgments is suitable when the group acts as one individual and the opinions of the decision makers are explicitly exchanged. Aczel and Alsina [30] pointed out that the weighted geometric mean (WGM) operator is the only appropriate method for the aggregation of individual judgments when the weights of the decision makers are not equal.

Bernasconi et al. [31] investigated the empirical properties of the various aggregation methods of aggregating individual judgments and individual priorities in group decision making. In the Atanassov's intuitionistic fuzzy circumstances, different types of aggregation methods and operators have been proposed to fuse the Atanassov's intuitionistic fuzzy preference information, such as the intuitionistic fuzzy weighted averaging (IFWA) operator [22,32], the intuitionistic fuzzy weighted geometric (IFWG) operator [33], the symmetric intuitionistic fuzzy weighted geometric (SYAIFWG) operator [34] and so on [35,36]. Choosing an appropriate aggregation operator to fuse the group preference information is very important.

It is well known that if all the individual MPRs are of perfect consistency, their weighted geometric MPR is of perfect consistency [1,6]. Xu [37] further proved that if all the individual MPRs are of acceptable consistency, their weighted geometric MPR is also of acceptable consistency. Although Lin et al. [38] questioned about Xu's conclusion, Grošelj and Stirn [39] further provided another strict proof for this conclusion. This conclusion is very attractive because it implies that once all the individual MPRs pass the consistency test then the group MPR derived by the WGM operator would pass the consistency test as well and there is no need to check it. In fact, this property has been applied widely in group decision making [40–43]. As for group decision making with IFPRs, whether this conclusion still holds or not with the intuitionistic fuzzy aggregation operators is a question. Recently, based on Xu et al.'s [20] definition of multiplicative consistency, Xu and Xia [44] proved that if all individual IFPRs are perfect multiplicative consistent, then the fused IFPR aggregated by the SIFWA operator is perfect multiplicative consistent. This work can be seen as the first attempt to answer the question. However, their study has several flaws:

- (1) As presented by Liao and Xu [21], Xu et al.'s [20] definition of multiplicative consistent IFPR is unreasonable, and thus Xu and Xia's [44] conclusion is somehow not reliable.
- (2) The conclusion of Xu and Xia [44] only reveals that the perfect multiplicative consistency of the aggregated IFPR under the condition that all individual IFPRs are perfect multiplicative consistent. If some of the individual IFPRs are not perfect multiplicative consistent but only acceptable multiplicative consistent, whether the conclusion still holds or not is a question.

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