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# Trust based consensus model for social network in an incomplete linguistic information context

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ABSTRACT

A theoretical framework to consensus building within a networked social group is put forward. This article investigates a trust based estimation and aggregation methods as part of a visual consensus model for multiple criteria group decision making with incomplete linguistic information. A novel trust propagation method is proposed to derive trust relationship from an incomplete connected trust network and the trust score induced order weighted averaging operator is presented to aggregate the orthopairs of trust/distrust values obtained from different trust paths. Then, the concept of relative trust score is defined, whose use is twofold: (1) to estimate the unknown preference values and (2) as a reliable source to determine experts' weights. A visual feedback process is developed to provide experts with graphical representations of their consensus status within the group as well as to identify the alternatives and preference values that should be reconsidered for changing in the subsequent consensus round. The feedback process also includes a recommendation mechanism to provide advice to those experts that are identified as contributing less to consensus on how to change their identified preference values. It is proved that the implementation of the visual feedback mechanism guarantees the convergence of the consensus reaching process.


## 1. Introduction

Social network analysis [25,47,50] studies the relationships between social entities like members of a group, corporations or nations. Of particular interest, and the focus of this paper, is to investigate consensus building between a group of experts connected via a network in which they explicitly express opinions in the form of trust and distrust statements, which are referred herein as trust network [48] and orthopairs of trust/distrust values [16], respectively. By analysing trust relationships between the networked experts, the concept of trust score and knowledge deficit are defined and used to propose an order relation on the set of orthopairs of trust/distrust values, which will be used to distinguish the most trusted expert from the group and, ultimately, to drive the aggregation of the individual opinions in order to arrive at a group consensual decision making solution.

In a general multiple criteria group decision making (MCGDM), a group of experts express preference values on alternatives under multiple criteria and interact to derive a common solution [35]. Experts usually come from multiple organisations and/or may have different backgrounds and knowledge on the decision making problem faced. Over the past decades, a large number of researchers have been attracted into this field [8–10,17,29–31,34,65]. These proposed models have been developed under the assumption that the preference values on alternatives under multiple criteria are completely expressed by experts. However, this assumption may not be completely realistic due to lack of in-depth knowledge of the problem domain by all or some of experts [32,45]. Thus, a key issue that needs to be addressed in this type of decision making environment is that of 'estimating unknown preference values'. There exist algorithms available to estimate unknown preference values in decision making based on the notion of consistency but not in trust [1–4,7,12,13,27,41], which is a new key knowledge that is possible to find in social network [33,46]. An objective of this paper is to develop a social trust based estimation method for MCGDM with incomplete preferences. On the other hand, another key issue in this type of decision making problem is how to reach consensus to derive the decision solution [6,28]. The interactive consensus model is regarded as an effective method to reach satisfactory consensus level because it implements a feedback mechanism to advice experts on how to change their preferences [5,19,23,24,54,57,64]. The known interactive consensus models force experts to change their preference values when consensus is below a threshold value. However, this may conflict with decision making in real practice because it is up to the experts to implement or not the given recommendations [22,53]. Additionally, these consensus models have the limitation that there is no visual representation to help them analyse their consensus position within the group.

The aim of this paper is to present a new trust based consensus model for social network in a 2-tuple linguistic context [20,21,26,38] under incomplete information. In this model, a policy allowing experts to revisit their evaluations using appropriate and meaningful consensus information representation within the social network framework is implemented. Firstly, a novel social trust propagation method to derive unknown information associated to an expert using trusted third partners (TTPs) is proposed. The trust score induced order weighted averaging (TS-IOWA) operator is developed to aggregate the orthopairs of trust/distrust values obtained from different trust paths. Secondly, a novel visual feedback process for MCGDM is designed to provide experts with: (1) visual representations of their consensus status within the group and (2) individual advice on how to change preference values.

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Additionally, visual simulation of future consensus status is generated to support experts in revisiting their evaluations and make changes to achieve a higher level of consensus. When this visual feedback mechanism is used to guide the consensus reaching process then its convergence is guaranteed within the social network.

The rest of paper is set out as follows: Section 2 introduces the trust network and an order relation of orthopairs of trust/distrust values, as well as the novel trust propagation and trust aggregation operators. In Section 3, the concept of relative trust score (RTS) and average trust degree (ATD) are defined. The first degree is used to estimate the unknown 2-tuple linguistic preference values, while the second one is used to aggregate the individual 2-tuple linguistic preference relations. Section 4 presents a new visual consensus model for social network that integrates visual representations of experts' consensus status within the group, the identification of experts and preference values that contribute less to consensus, individual advice on how to change preference values and visual simulation of future consensus status. The convergence of the consensus reaching process is also proved when this visual feedback mechanism is used to guide it. An analysis of the proposed visual consensus model with respect to existing consensus models in literature is given in Section 5. Finally, conclusions are drawn in Section 6.

## 2. Trust propagation and aggregation in social network

Social network analysis (SNA) [25,47,50] studies the relationships between social entities like members of a group, corporations or nations. Therefore, it enables us to examine their structural and locational properties including centrality, prestige, structural balance, trust relationship etc. There are three notational schemes in SNA analysis: set of actors, the relations themselves, and the actor criteria (see Table 1). As a consequence, we can refer to important network concepts in a unified manner.

- Graph theoretic: in which the network is viewed as a graph consisting of nodes joined by lines.
- Algebraic: this notation presents the advantage that allow us to distinguish several distinct relations and represent combinations of relations.
- Sociometric: in which relational data are often presented in two-ways matrices called sociomatrix.

The above sociomatrix is a binary or crisp relation. However, in many situations, it may not be suitable to represent the relation in a crisp way because it is not clear cut defined. Notice that in real life too, trust is often interpreted as a gradual concept as humans do not just reason in terms of 'trusting' and 'not trusting', but rather trusting someone 'very much' or 'more or less' [18]. Victor et al. [48] introduce the following adapted bilattice structure based on the use of orthopairs of trust/distrust values as follows:

**Definition 1.** The set of orthopairs of trust/distrust values ($BL^{\square}$) can be endowed with a bilattice structure with the following trust ordering ($\leq_t$), knowledge ordering ($\leq_k$), and negation operator ($\neg$):

$$BL^{\square} = ([0,1]^2, \leq_t, \leq_k, \neg)$$

$$(t_1, d_1) \leq_t (t_2, d_2) \text{ iff } t_1 \leq t_2 \text{ and } d_1 \geq d_2$$

$$(t_1, d_1) \leq_k (t_2, d_2) \text{ iff } t_1 \leq t_2 \text{ and } d_1 \leq d_2$$

$$\neg(t_1, d_1) = (d_1, t_1)$$

As stated by Victor et al. [48], it is clear that 'the lattice ($[0,1]^2, \leq_t$) orders the [orthopairs of trust/distrust values] going from complete distrust (0, 1) to complete trust (1, 0), [while] the ($[0,1]^2, \leq_k$) evaluates the amount of available trust evidence, ranging from [...] incomplete information [$t_1 + d_1 < 1$] to [...] inconsistent or contradictory information [$t_1 + d_1 > 1$]'. Thus, two orthopairs of trust/distrust values with same trust values and different distrust values will be placed in reverse order by the trust ordering $\leq_t$ and the knowledge ordering $\leq_k$, and thus to avoid this outcome Victor et al.'s ordering approach will not be used. Having said this, it is noticed here that the methodology used in the case of intuitionistic fuzzy sets [52,55] for the concepts of score and accuracy of an intuititionistic value can be applied in this context to define the following concepts of trust score and knowledge deficit so that a two steps complete ordering on the set of orthopairs of trust/distrust values can be derived [48]:

**Definition 2** *(Trust score and knowledge deficit)*. The trust score and knowledge deficit associated to an orthopair of trust/distrust values $(t_1, d_1)$ are:

$$TS(t_1, d_1) = t_1 - d_1$$

$$KD(t_1, d_1) = |1 - t_1 - d_1|.$$

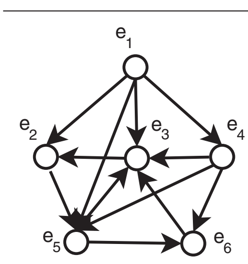
Following Victor et al. [48], we say that orthopairs of trust/distrust values $(t_1, d_1)$ for which $KD(t_1, d_1) = 0$, i.e., $t_1 + d_1 = 1$, have perfect knowledge (i.e., complete trust state), while all others will have a deficit in knowledge. The combination of both trust score and knowledge deficit can be used to propose the following order relation for the set of orthopairs of trust/distrust values.

**Definition 3.** Let $(t_1, d_1)$ and $(t_2, d_2)$ be orthopairs of trust/distrust values, $TS_1 = t_1 - d_1$ and $TS_2 = t_2 - d_2$ their associated trust scores, and $KD(t_1, d_1) = |1 - t_1 - d_1|$ and $KD(t_2, d_2) = |1 - t_2 - d_2|$ their associated knowledge deficits, respectively. We have that

1. If $TS_1 < TS_2$, then $(t_1, d_1)$ is smaller than $(t_2, d_2)$, denoted by $(t_1, d_1) < (t_2, d_2)$;
2. If $TS_1 > TS_2$, then $(t_1, d_1)$ is greater than $(t_2, d_2)$, denoted by $(t_1, d_1) > (t_2, d_2)$;
3. If $TS_1 = TS_2$, then:
   (a) if $KD(t_1, d_1) < KD(t_2, d_2)$, then $(t_1, d_1)$ is greater than $(t_2, d_2)$, denoted by $(t_1, d_1) > (t_2, d_2)$;
   (b) if $KD(t_1, d_1) > KD(t_2, d_2)$, then $(t_1, d_1)$ is smaller than $(t_2, d_2)$, denoted by $(t_1, d_1) < (t_2, d_2)$;
   (c) if $KD(t_1, d_1) = KD(t_2, d_2)$, then $(t_1, d_1)$ is equal to $(t_2, d_2)$, denoted by $(t_1, d_1) = (t_2, d_2)$.

Thus, when comparing two orthopairs of trust/distrust values, the one with higher trust score is ordered first, and in case of equal trust scores, the lower knowledge deficit prevails. We can utilise the order relation of orthopairs of trust/distrust values to distinguish the most trusted expert from a group or path in a trust network, which is useful in fusing individual opinions because it can be used to induce the ordering of an OWA based aggregation of the decision matrix values. This will be elaborated later in the paper.

**Table 1**
Different notations in social network analysis.




Graph

Algebraic:

| | |
|---|---|
| $E_1RE_2$ | $E_4RE_3$ |
| $E_1RE_3$ | $E_4RE_5$ |
| $E_1RE_4$ | $E_4RE_6$ |
| $E_1RE_5$ | $E_5RE_3$ |
| $E_2RE_5$ | $E_5RE_6$ |
| $E_3RE_2$ | $E_6RE_3$ |

Sociometric:

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$