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Approximating source location and star survivable network problems [☆]

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ABSTRACT

In Source Location (SL) problems the goal is to select a minimum cost source set $S \subseteq V$ such that the connectivity (or flow) $\psi(S,v)$ from S to any node v is at least the demand d_v of v. In many SL problems $\psi(S,v)=d_v$ if $v \in S$, so the demand of nodes selected to S is completely satisfied. In a variant suggested recently by Fukunaga [7], every node v selected to S gets a "bonus" $p_v \le d_v$, and $\psi(S,v)=p_v+\kappa(S \setminus \{v\},v)$ if $v \in S$ and $\psi(S,v)=\kappa(S,v)$ otherwise, where $\kappa(S,v)$ is the maximum number of internally disjoint (S,v)-paths. While the approximability of many SL problems was seemingly settled to $\Theta(\ln d(V))$ in [20], for his variant on undirected graphs Fukunaga achieved ratio $O(k \ln k)$, where $k=\max_{v \in V} d_v$ is the maximum demand. We improve this by achieving ratio $\min\{p^* \ln k, k\} \cdot O(\ln k)$ for a more general version with node capacities, where $p^*=\max_{v \in V} p_v$ is the maximum bonus. In particular, for the most natural case $p^*=1$ we improve the ratio from $O(k \ln k)$ to $O(\ln^2 k)$. To derive these results, we consider a particular case of the Survivable Network (SN) problem when all edges of positive cost form a star. We obtain ratio $O(\min\{\ln n, \ln^2 k\})$ for this variant, improving over the best ratio known for the general case $O(k^3 \ln n)$ of Chuzhoy and Khanna [4]. Finally, we obtain a logarithmic ratio for a generalization of SL where we also have edge-costs and flow-cost bounds $\{b_v : v \in V\}$, and require that the minimum cost of a flow of value d_v from S to every node v is at most b_v.

1. Introduction

In Source Location (SL) problems, the goal is to select a minimum cost source set $S \subseteq V$ such that the connectivity from S to any node v is at least the demand d_v of v. Formally, the generic version of this problem is as follows.

> Source Location (SL)
> *Instance:* A graph $G=(V,E)$ with node-costs $c=\{c_v : v \in V\}$, **connectivity demands** $d=\{d_v : v \in V\}$, and a **source connectivity function** $\psi : 2^V \times V \to \mathbb{Z}_+$, where $\mathbb{Z}_+$ denotes the set of non-negative integers.
> *Objective:* Find a minimum cost source node set $S \subseteq V$ such that $\psi(S,v) \ge d_v$ for every $v \in V$.

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Table 1
Previous approximation ratios and lower bounds for SL problems. GC and UC stand for general and uniform costs, GD and UD stand for general and uniform demands, respectively.

c,d	λ $(p,q \equiv k)$		κ	
	Undirected	Directed	Undirected	Directed
GC,GD	$\Theta(\ln d(V))$ [2,20]	$\Theta(\ln d(V))$ [2,20]	$\Theta(\ln d(V))$ [2,20]	$\Theta(\ln d(V))$ [2,20]
GC,UD	in P [1]	$O(\ln d(V))$ [2]	$O(\ln d(V))$ [2]	$O(\ln d(V))$ [2]
UC,GD	in P [1]	$O(\ln d(V))$ [2]	$O(\ln d(V))$ [2]	$O(\ln d(V))$ [2]
UC,UD	in P [22]	in P [3]	$O(\ln d(V))$ [2]	$O(\ln d(V))$ [2]
	$\hat{\kappa}$ $(p \equiv k, q \equiv 1)$		κ' $(q \equiv 1)$	
GC,GD	$\Theta(\ln d(V))$ [20]	$\Theta(\ln d(V))$ [20]		
	$O(k \ln k)$ [7]		$O(k \ln k)$ [7]	
GC,UD	in P [16]	in P [16]		
UC,GD	$O(\ln d(V))$ [20]	$O(\ln d(V))$ [20]		
	$O(k)$ [9]			
UC,UD	in P [16]	in P [16]		

Several source connectivity functions ψ appear in the literature. To avoid considering many cases, we suggest two generic types, that include previous particular cases.

Definition 1.1. An integer set-function f on a groundset U is **submodular** if $f(A)+f(B) \geq f(A \cap B)+f(A \cup B)$ for all $A, B \subseteq U$, and f is **non-decreasing** if $f(A) \leq f(B)$ for all $A \subseteq B \subseteq U$.

Definition 1.2. Let $G=(V,E)$ be a graph with node capacities $\{q_u : u \in V\}$. For $S \subseteq V$ and $v \in V$ the (S,v)**-**q**-connectivity** $\lambda^q_G(S,v)$ is the maximum number of edge-disjoint paths from $S \setminus \{v\}$ to v in G such that every node $u \in V$ is an internal node in at most q_u paths. Given **connectivity bonuses** $\{p_u \geq q_u : u \in V\}$, the (S,v)-(p,q)**-connectivity** $\lambda^{p,q}_G(S,v)$ is defined by: $\lambda^{p,q}_G(S,v)=p_v+\lambda^q_G(S,v)$ if $v \in S$, and $\lambda^{p,q}_G(S,v)=\lambda^q_G(S,v)$ otherwise.

We will say that a source connectivity function $\psi(S,v)$ is **submodular** if for every $v \in V$ the function $f_v(S)=\psi(S,v)$ is submodular and non-decreasing; $\psi(S,v)$ is **survivable** if it is of the type $\psi(S,v)=\lambda^{p,q}_G(S,v)$. The concept of q-connectivity is essentially "mixed connectivity" (the case $q_u \in \{0,k\}$) introduced by Frank, Ibaraki, and Nagamochi [5], while (p,q)-connectivity combines it with the connectivity function introduced recently by Fukunaga [7] (the case $q \equiv 1$). The case of arbitrary node capacities includes additional connectivity versions compared to [7], e.g., the edge-connectivity case.

It is not hard to see that every survivable source connectivity function $\psi(S,v)$ is submodular (see Section 4), but the inverse is not true in general. This gives only two types of SL problems.

Submodular SL: The connectivity function $\psi(S,v)$ is submodular.
Survivable SL: The connectivity function $\psi(S,v)$ is survivable.

We list four source connectivity functions that appear in the literature. All of them are submodular, and three of them are also survivable. Given an SL instance let $k=\max_{v \in V} d_v$ denote the maximum demand, and in the case of Survivable SL let $p^*=\max_{u \in V} p_u$ denote the maximum connectivity bonus and $q^*=\min_{u \in V} q_u$ denote the minimum node capacity. In what follows assume that $1 \leq q_u \leq p_u \leq k$ for all $u \in V$, and thus $1 \leq p^* \leq k$ and $1 \leq q^* \leq k$ holds.

1. λ-SL: $\lambda_G(S,v)$ is the maximum number of pairwise edge-disjoint (S,v)-paths if $v \notin S$ and $\lambda_G(S,v)=\infty$ otherwise. This is Survivable SL with $p_u=q_u=k$ for every $u \in V$.
2. κ-SL: $\kappa(S,v)$ is the maximum number of (S,v)-paths no two of which have a common node in $V \setminus (S \cup v)$ if $v \notin S$, and $\kappa(S,v)=\infty$ otherwise.
3. $\hat{\kappa}$-SL: $\hat{\kappa}(S,v)$ is the maximum number of (S,v)-paths no two of which have a common node in $V \setminus \{v\}$ if $v \notin S$, and $\hat{\kappa}(S,v)=\infty$ otherwise. This is Survivable SL with $p_u=k$ and $q_u=1$ for every $u \in V$.
4. κ'-SL: $\kappa'(S,v)=\hat{\kappa}(S,v)$ if $v \notin S$ and $\kappa'(S,v)=p_v+\hat{\kappa}(S \setminus \{v\},v)$ if $v \in S$. This is Survivable SL with $q_u=1$ for every $u \in V$.

The known approximability status of SL problems with source connectivity functions $\lambda,\kappa,\hat{\kappa},\kappa'$, is summarized in Table 1; see also a survey in [15]. The approximability of $\lambda,\kappa,\hat{\kappa}$-SL problems was settled to $O(\ln d(V))$ in [20] (where $d(V)=\sum_{v \in V} d_v$), while Fukunaga [7] showed that undirected κ'-SL admits ratio $O(k \ln k)$. We prove the following.

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