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Conditional diagnosability of a class of matching composition networks under the comparison model ☆,☆☆

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ABSTRACT

Fault diagnosis of interconnection networks is an important consideration in the design and maintenance of multiprocessor systems. Herein, we study fault diagnosis, which is the identification of faulty processors in high speed parallel processing systems. Conditional diagnosability, proposed by Lai et al. [22], assumes that no fault set can contain all the neighbors of any processor in a system; this is a well-accepted and general measure of the diagnosis ability of an interconnection network of multiprocessor systems. The diagnosability and conditional diagnosability of many interconnection networks have been studied using various diagnosis models. In this paper we study the conditional diagnosability of matching composition networks under the comparison model (MM* model). In [31] Yang determined a set of sufficient conditions for a network G to be conditionally $(3n-3-C(G))$-diagnosable. Our main contribution in this paper is to extend Yang's result by determining a larger class of networks that are conditionally $(3n-3-C(G))$-diagnosable. Yang's result [31] and earlier results for the hypercube, the crossed cube, the twisted cube and the Möbius cube [18,32,33] all become corollaries of our main result. Thus this paper extends the state of the art in the area of conditional diagnosability of multiprocessor systems.

1. Introduction

Continuous advances in semiconductor technology have made it possible to develop very large digital systems comprising of hundreds of thousands of components or units. Yet, it is impossible to build such systems without defects. As the size of a system increases, it is more likely to develop faults both in the manufacturing process as well as during the operation period. Testing of such systems becomes extremely difficult owing to their large sizes. First, the complexity of test generation for such large systems is overwhelming. Second, the application of test data, as well as the observation and analysis of test responses is extremely difficult and costly, even if test data for the same can be generated. This problem may be

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further aggravated by the likely geographical distribution of units. Testing of such systems based on the traditional stimuli-supplying and responses-observing philosophy has become virtually impossible. In 1967, Preparata, Metze, and Chien [26] proposed a model and a framework, called System-Level Diagnosis, for addressing this problem. In the more than four decades following this pioneering work, several issues arising from the application of this framework have been investigated and resolved. Many of these results have profound theoretical and practical implications. Most of the recent research efforts in system-level diagnosis have focused on enhancing the applicability of system-level diagnosis based approaches to practical scenarios. In particular, the focus has been on:

1. Probabilistic diagnosis and application to VLSI testing.
2. On-line distributed diagnosis and application to the diagnosis of a networked cluster of workstations.

Examples of important advances in system-level diagnosis and applications may be found in [4–11,14,16,17,19,20,26,29].

The focus of this paper is on system diagnosis, which involves locating faulty processors. One of the most popular models in dealing with this problem is the *PMC model*, which was proposed by Preparata, Metze, and Chien [26] in 1967. In the PMC model, a test involves a pair of adjacent processors: the testing and the tested processor. It is also assumed that a test result is reliable if and only if the testing processor is not faulty. Since the introduction of the PMC model, many of its variants have been proposed. Among them, two are particularly relevant in our context: the symmetric comparison model of Chwa and Hakimi [5] and the asymmetric comparison model of Malek [25]. The two models assume the existence of a central observer that collects information about comparisons and then performs a diagnosis of the system. The difference between the two models lies in the different assumptions about the comparison results for two faulty processors. In the symmetric model, it is assumed that the outputs of two compared processors may be the same if they are both faulty while in the asymmetric model it is assumed that the outputs of two such processors are always different. Since for a complex computation task, identical errors for two faulty processors are rare, the asymmetric comparison model is more realistic.

Another model, proposed by Maeng and Malek [24], the MM model, assumes that comparisons are executed by the processors themselves (processors adjacent to both of the two compared ones) and only comparison results are sent to the central observer, which then completes the diagnosis of the system. Maeng and Malek [24] also presented a special case of the MM model, called the MM* model, in which a processor executes comparisons for any pair of its neighboring processors. MM* model is the diagnosis model studied in this paper. Let us describe this model in detail. A graph $G = (V(G), E(G))$ is used to represent a system where each vertex represents a processor and each edge represents a link. Assign a task to each vertex. The vertex w is a comparator of a pair of processors $\{u, v\}$ if $(u, w) \in E(G)$ and $(v, w) \in E(G)$. The outcome of this comparison is denoted by $\sigma((u, v)_w)$ where

$$\sigma((u,v)_w) = \begin{cases} 0, & \text{if } \{u, v, w\} \cap F = \emptyset \\ 1, & \text{if } w \notin F \text{ and } \{u, v\} \cap F \neq \emptyset \\ 0 \text{ or } 1, & \text{if } w \in F \end{cases}$$

where F is the set of faulty processors.

The set of all comparison outcomes is called a syndrome σ of the system. For a given syndrome σ, a subset of vertices $F \subseteq V(G)$ is said to be *consistent* with σ if syndrome σ can be produced when the faulty set of G is F. The comparison result is 0 or 1 when the comparison is performed by a faulty comparator. Therefore, on one hand, a faulty set F may produce a number of different syndromes. On the other hand, different faulty sets may produce the same syndrome. Define $\sigma_F = \{\sigma \mid F \text{ is consistent with } \sigma\}$. Two distinct sets F_1, F_2 belonging to $V(G)$ are said to be *indistinguishable* if $\sigma_{F_1} \cap \sigma_{F_2} \neq \emptyset$; otherwise, F_1, F_2 are said to be *distinguishable*. A system is said to be *t-diagnosable* if, given a syndrome σ, there is a unique set of faulty vertices that is consistent with σ while the number of faulty vertices does not exceed t. The *t-diagnosability problem* is to determine the largest value of t for which a system G is t-diagnosable. Considering classical measures of diagnosability for multiprocessor systems under the comparison model, if all neighbors of a processor v are simultaneously faulty, it is impossible to determine whether the processor v is fault-free or faulty. Therefore, the diagnosability of a system is limited by its minimum vertex degree. For practical systems, the probability that all neighbors of a vertex are simultaneously faulty is very low. Owing to this reason, Lai et al. [22] proposed a new measure of diagnosability as described in the following. A fault set $F \subset V(G)$ is called a *conditional faulty set* if $N_G(v) \nsubseteq F$ for any vertex $v \in V(G)$, where $N_G(v)$ is the set of neighbors of v in G. Two distinct conditional faulty sets F_1, F_2 belonging to $V(G)$ are said to be an indistinguishable conditional pair if $\sigma_{F_1} \cap \sigma_{F_2} \neq \emptyset$; otherwise, F_1, F_2 are said to be a *distinguishable conditional pair*. A system $G = (V(G), E(G))$ is *conditionally t-diagnosable* if any pair of conditional faulty sets F_1, F_2 with $F_1 \neq F_2$, $|F_1| < t$ and $|F_2| < t$ are distinguishable. The *conditional diagnosability* of a system G, denoted as $t_c(G)$, is defined to be the maximum value of t such that G is conditionally t-diagnosable.

A multiprocessor system can be modeled by an undirected simple graph with nodes and links modeled as vertices and edges, respectively. The graph used to model the multiprocessor system is called the interconnection network of the multiprocessor system. Choosing an appropriate interconnection network is important for a system's design and maintenance. Therefore, the analysis of the properties of interconnection networks is an important research topic in high-performance computing.

The hypercube [27] is one of the most popular interconnection networks, and many of its properties have been studied in the literature. Crossed cubes [12], Möbius cube [23], and twisted cubes [15] are several variations of hypercubes. These variants of hypercubes preserve many of the good properties of hypercubes such as high symmetry. At the same time, their diameter is about a half of a hypercube of the same size. Thus, these interconnection networks are regarded as good

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