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# On consensus models with utility preferences and limited budget

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ABSTRACT

This paper mainly focuses on the consensus problem with utility preferences denoted by simple trapezoidal membership function. In group decision-making (GDM), for acquiring the best consensus opinion, not only the total cost required by achieving the consensus, but also the utility of all the decision-makers (DMs) should be considered. Ben-Arieh et al. propose a consensus model from the view of the minimized cost. Based on their models, a kind of optimization consensus model has been put forward under the constraints of limited budget and different kinds of utility, whose objective function aims to obtain the maximum utility level of the whole GDM process. From an economic point of view, results show that different utility preferences of all the individual DMs have impacts on the final optimal consensus opinion. Besides, the moderator has a dominant role in the development trends throughout the whole decision-making process to some extent. Numerical examples are given to deeply explain the proposed models.



## 1. Introduction

Group consensus is the prerequisite for effective decision-making. Rapid development of the Internet and communication technology makes it possible for large group decision-making (GDM) with different regions, motivations or educational backgrounds. Also, it further improves the reliability of the decision-making results. However, large GDM may reduce the efficiency of reaching a consensus [1], and increase the consumption of resources (e.g., time, manpower or cost), which will definitely increase the total cost during the decision-making process. As a result, how to obtain a consensus with the minimum cost becomes one of the hot topics for the researchers. In 2007, Ben-Arieh and Easton [2] proposed the minimum cost consensus models based on distance measure method, and they found the optimal consensus points for both rectilinear and squared geometric cost functions. Then in 2009, they [3] further explore the above consensus models with/without a threshold and build a model for solving the maximum number of experts with budget constraint. In addition, their models are all deeply investigated by introducing the concept of “aggregation operators” (see Refs. [4,5]).

Based on aggregation method, Wu et al. [6,7] believe that reaching a consensus in GDM is actually driven by trust between experts in the social network environment. Also, in their trust-based consensus models, consensus degree/level is an important index to guide the feedback mechanism. Generally, two main approaches (i.e., hard consensus measure and soft consensus measure) are usually adopted to measure the degree of consensus in the above models: the former varies between 0 (no consensus or partial consensus) and 1 (full consensus), while the latter deals with a more flexible GDM environment [8]. So, soft consensus is more consistent with the real decision-making situations, and therefore it’s more favored by scholars. Through large amount of pioneering and prominent papers, Herrera-Viedma et al. [9] present an overview of consensus models based on soft consensus measures, and they conclude the main approaches, the new trends and two important challenges. Relatively, Cabrerizo et al. [8] put more emphasis on the consensus approaches in fuzzy GDM problems to compute soft consensus measures as well as their advantages and drawbacks. To the best of our knowledge, the existing research on consensus models are mostly concerning the following aspects:

- Consensus models in Web framework: Web is characterized by large user base and real-time communication, which brings diversity opinions, and also the difficulty of convergence in GDM. To help users reach decision with a high level of consensus in such virtual environments, a linguistic consensus model with

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delegation scheme and feedback mechanism is proposed by Alonso et al. [10]. In fact, their research can be regarded as an extension of [11], which presents a web based consensus support system to solve GDM problems with three kinds of incomplete preference relations. Differ from Alonso's research, Pérez et al. [12] build a consensus model with fuzzy ontology to deal with the psychology in negotiation process, while [13] incorporates mobile technologies to provide decision support for dynamic decision environments.

- Consensus models for heterogeneous GDM: Classical models usually view all experts' opinions equally important, or use weighted aggregation operators to deal with the heterogeneous decision context. Pérez et al. [14] introduce a new feedback mechanism that adjusts the advice amount based on each expert's own relevance or importance level, and their model makes the concept of importance guide the whole GDM process, rather than the moderator. Moreover, Chandan and Debjani [1] propose a fuzzy clustering methodology to avoid the heterogeneity among the opinions, i.e., they provide an automatic decision-aid tool for heterogeneous GDM problems.
- Consensus models with incomplete fuzzy preference: Cabrerizo et al. [15] use consistency and consensus measures, and apply a feedback mechanism to support the management of incomplete unbalanced fuzzy linguistic information in GDM problems. Compared with Cabrerizo's research, Wu and Chiclana [16] make the feedback mechanism visualized and take experts' risk attitudes into account when they explore the GDM with triangular fuzzy complementary preference. Other work in this field can be found in [17–19], in which the entire GDM process are all guided by some specific indexes and has feedback mechanism to help experts change their preferences to achieve a higher consensus level.

Without a doubt, reaching a high consensus level is the basic aim of all consensus models, but it's also necessary to consider each individual DM's utility: every DM hopes to get enough attention with their interest highly valued, as he/she has a significant impact on the consensus reached. Thus, each individual's utility (preference) must be taken into account by the moderator [2,3,20]. Actually, the introduction of DMs' utility plays a huge role in GDM research. Chang et al. [21–24] have done a lot of work, for example, they use S-shaped penalty function or binary piecewise linear membership function, or by setting multiple aspiration levels to greatly improve the utility level of goal programming. Meanwhile, to overcome the defect that the value of DMs' opinions may be probabilistic or not clear, Aouni et al. [25,26] exploit utility/satisfaction function to describe the DM's opinion preference based on the stochastic goal programming model or the type of deviation. Besides, Yang and Sen [27] design a method that can capture DM's opinion utility, and dig out the best compromising alternative by using utility function. Most importantly, the concept of utility has also acquired a large development in practical application, e.g., the house selection via the Internet [28] and the transportation planning [29]. Since utility (or membership) function can well represent DM's utility, that is, they can more realistically simulate the actual decision-making scenarios, and current consensus models rarely consider DM's utility, we introduce the concept of utility into the consensus models with the help of simple trapezoidal membership function.

In addition, available consensus models either only consider the cost of decision-making, or just explore the utility of DM's preferences. Only considering the total cost of decision-making without the individual utility constraints, will hardly reflect the DMs' value in GDM. By the same token, only exploring the decision-making utility level without looking into the cost of consensus attained, will barely clarify the effect of resource consumption in consensus decision-making. Therefore, in this research, we explore the consensus problems in GDM by incorporating both the consensus cost and DMs' utility. In other words, how to maximize the utility of GDM under the limited cost is the basic target of this paper. Besides, due to the moderator represents the leader of the entire negotiation process and he/she always has deeper knowledge about the specific GDM problem [14], which make he/she have enough authority to persuade the others to reach an agreement. Therefore, this paper views the moderator as an independent decision-making unit, who also has different utility preferences just like all the individual DMs. Furthermore, we also investigate the practical significance of consensus model with utility constraints from an economic point of view.

The remainder of this paper is organized as follows. In Section 2, the minimum cost consensus model is firstly introduced, and the interval opinions of DMs as well as the concept of utility are proposed. Section 3 focuses on the construction of two kinds of optimization consensus models with different utility preferences of DMs under limited budget constraint. Furthermore, the economic significance of these models are also explored in this section. In order to further explain the proposed models, numerical examples are included in Section 4. Finally, the concluding remarks and the future work are presented in Section 5.

## 2. Problem description

Suppose there are $m$ DMs that take part in GDM, and the set of all DMs is denoted by $D=\{d_1, d_2, \ldots, d_m\}$. Let $o_i$ ($i \in M=\{1, 2, \ldots, m\}$) represent the opinion of the $i-th$ DM, and $o'$ be the optimal group consensus that will be obtained after several rounds of consultations between the moderator and all the DMs. Without loss of generality, the role of a moderator is just similar to a leader in an organization, who has good communication and negotiation skills, and can influence the development trend of the entire GDM. In fact, obtaining the optimal consensus opinion $o'$, the moderator often needs to pay a lot of resources (such as time, money etc.) to persuade DMs to change their opinions. So, from the moderator's view, he/she hopes to achieve the best consensus opinion by paying the minimum cost to all the individuals.

In general, the calculation of consensus index in a GDM is usually done by measuring the deviations between the individual opinions and the group consensus opinion [5]. Obviously, the less the deviations are, the higher degree of consensus is. In some complex GDMs, the moderator hopes to increase the degree of consensus by consuming more resources (or paying more costs). The optimal consensus model, based on a minimum cost, proposed by Ben-Arieh and Easton [2] in 2007 can be described as:

$$\begin{aligned} &\min \sum_{i=1}^{m} c_i|o_i - o'_i| \\ &s.t.\ |o_i - o'_i| \le \varepsilon, i = 1, 2, \ldots, m \end{aligned} \tag{1}$$

Where $c_i$ is the cost of DM $d_i$ for changing one unit opinion, $o'_i$ is the adjusted opinion of $d_i$, and the optimal solution $o'$ of the model is the final group consensus opinion. In 2009, Ben-Arieh et al. [3] put forward the minimum cost consensus model based on quadratic cost functions, which changed the above objective function into $\sum_{i=1}^{n} c_i(o' - o_i^0)^2$, (where $o_i^0$ represents the original opinion of the $i-th$ DM). By improving the model, they proposed three methods to explore the different mechanism of consensus reaching in GDM.

Model (1) can be transformed into a simpler form of linear programming, which makes it more convenient to solve. However, Model (1) only explores the acquisition of the optimal consensus with the least resources consumption paid by the moderator. Actually, it is necessary to take the interests of the individual into