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Item bidding for combinatorial public projects ☆

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ABSTRACT

We analyze a simple mechanism for the Combinatorial Public Project Problem (Cppp). The problem asks to select k out of m available items, so as to maximize the social welfare for autonomous agents with combinatorial preferences (valuation functions) over subsets of items. The Cppp constitutes an abstract model for decision making by autonomous agents and has been shown to present severe computational hardness, in the design of tractable truthful approximation mechanisms. We study a non-truthful mechanism that is, however, practically relevant to multi-agent environments, by virtue of its natural simplicity. The mechanism employs an item bidding interface, where every agent issues a separate bid for the inclusion of each distinct item in the outcome; the k items with the highest sums of bids are then chosen. As for the payment scheme, the agents are charged according to a direct adaptation of the VCG payment rule. For fairly expressive classes of the agents' valuation functions, we establish existence of socially optimal pure Nash equilibria, as well as strong equilibria, that are resilient to coordinated deviations of subsets of agents. Particularly with respect to pure Nash equilibria, we prove convergence of an iterative procedure. Subsequently, we derive worst-case bounds on the approximation of the optimum social welfare achieved in (strong) equilibrium by the mechanism. We show that the mechanism's performance improves with the number of agents that can coordinate their bids, and reaches half of the optimum welfare at strong equilibrium. Finally, we derive bounds on the mechanism's performance in Bayes–Nash equilibrium, under an incomplete information setting.

1. Introduction

Public project problems model situations where a central authority (e.g. a government or municipality) aims at carrying out a project in the common interest of all members of a community, such as building a bridge or a new road [22,23]. Several variations have been considered in the literature, motivated by different applications, see e.g., [24], Chapters 6–8. Our focus is on the *Combinatorial Public Project Problem* (Cppp), which was introduced by Papadimitriou, Schapira and Singer in [26] as a general prototypical model for decision making by autonomous strategic agents with *combinatorial preferences*. In the Cppp, an authority aims at combining at most k components from a given set of m distinct items, to build a composite service or facility, in favor of n strategic agents. Each agent values different subsets of items according to a *private valuation*

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Table 1
The ℓ-strong price of anarchy of the item bidding mechanism.

ℓ-Strong Price of Anarchy bounds		
Valuation functions	Upper	Lower
XOS	$1+\lceil \frac{n}{\ell} \rceil$	$\max\left\{2, \frac{n}{\ell}\right\}$
UD	$1+\min\left\{\lceil \frac{n}{\ell} \rceil, (1+\frac{n}{k\cdot\ell})\right\}$	$\max\left\{2, \frac{n}{k\cdot\ell}\right\}$

function, defined over all subsets of the m items. The problem then amounts to devising a *mechanism*, through which the authority will determine a *socially optimal outcome* – a welfare maximizing subset of items, along with the payments that the agents should issue for the outcome.

We analyze the performance of a simple mechanism for the Cppp, with respect to the approximation of the optimum social welfare that it achieves at equilibrium. The mechanism employs a simple *item bidding* interface for eliciting the agents' preferences, expressed as bids on individual distinct items, and a natural rule for outcome determination. Under the item bidding interface, each agent issues a separate *bid* for the inclusion of each distinct item in the outcome. In effect, each agent is forced to "compress" his combinatorial valuation function into an *additive* bid vector [5]. The mechanism that we study then selects the k items achieving the highest sums of bids. For determining the agents' payments, we use an adaptation of the familiar Vickrey–Clarke–Groves (VCG) pricing scheme (see e.g. [3]).

Item bidding mechanisms have received considerable attention in the recent literature, and especially in the context of combinatorial auctions [8,16,5,14,30,9]. Their appeal is due to the simple and natural bidding interface, turning them into a practical means that is already implicitly deployed in real-world online markets, as noted in [7,5]. On the negative side, our mechanism (and most other item bidding mechanisms) is vulnerable to manipulation, one of the reasons being the restricted expressiveness of the bidding interface. In order to alleviate the agents' strategic behavior, the field of mechanism design has traditionally advocated the implementation of *truthful* reporting of preferences in a *dominant strategies* equilibrium. However, most known truthful mechanisms for agents with combinatorial preferences consist of complex algorithmic schemes for determining the outcome and payments; thus their complexity hinders their practical deployment and discourages the agents from participating in the induced strategic game. Concerning the Cppp in particular, a series of works [26,29,6] have established severe computational inapproximability results for tractable truthful mechanisms. On the other hand, for quite expressive classes of the agents' valuation functions, the optimization problem underlying the Cppp is long known to be approximable within a constant factor, by the celebrated greedy algorithm of Nemhauser, Wolsey and Fischer [25]; hence, it is compelling to examine whether other than truthful mechanism models exist, with comparably favorable performance.

In order to assess the quality of our mechanism (and more generally of non-truthful mechanisms), we evaluate the performance of its equilibrium outcomes, in terms of the social welfare they achieve. Our work is along the same line as [18], where item bidding was paired with a natural "pay-as-bid" rule, to yield a "first-price" type of mechanism for the Cppp. The authors showed that their mechanism's performance improves when the agents can coordinate their bidding decisions. To this end, they proved favorable inefficiency bounds for *strong equilibria* [2], that are resilient to coordinated joint deviations of subsets of agents. We quantify our mechanism's inefficiency in a similar detailed fashion, by analyzing the inefficiency of ℓ-strong equilibria, to show that its performance improves gracefully with the "allowed" maximum size, ℓ, of subsets of agents that can coordinate.

1.1. Contribution

We describe a simple deterministic item bidding mechanism for the Cppp, and analyze its performance at equilibrium. For the fairly general class of *fractionally subadditive* valuation functions (also termed **XOS**), we prove existence of *socially optimal* pure Nash equilibria. In particular, we show that an iterative procedure converges to pure Nash equilibrium, while increasing the achieved welfare in each iteration. We also show that the item bidding mechanism admits strong equilibria for at least a smaller – yet expressive – class of *uniform Unit-Demand* valuation functions (**uUD**). These results signify the importance of employing our VCG-based pricing scheme; in contrast, the "pay-as-bid" rule used in [18], prevents existence of Nash equilibria in general, unless specific tie-breaking rules are employed.

Subsequently, we derive bounds on the worst case ratio of the socially optimal welfare over the welfare achieved by the mechanism at ℓ-strong equilibrium (see Table 1). This ratio was studied by Andelman, Feldman and Mansour in [1] under the term *ℓ-strong Price of Anarchy* and measures the mechanism's performance with respect to social welfare approximation at equilibrium. To analyze the ℓ-strong Price of Anarchy, we make a standard *no-overbidding* assumption [8,5,14,30,10] whereby agents never outbid their value for any subset of items. For agents with **XOS** valuation functions we prove an upper bound of $1+\lceil n/\ell \rceil$; we also give a lower bound of $\max\{2, n/\ell\}$, which almost matches our upper bound, and holds for agents with as simple as linear valuation functions. Let us note that, in order to derive our upper bound for agents with **XOS** valuation functions, we utilize a slightly generalized version of the *ℓ-coalitional smoothness* arguments derived recently by Bachrach et al. in [4], for upper bounding the strong Price of Anarchy in utility maximization games. In effect, we obtain that the strategic game induced by the item bidding mechanism is *coalitionally smooth*; by using the results of [4], this property allows us to bound the mechanism's inefficiency by $O(\log n)$, in every *coalitional Sink Equilibrium* that is eventually reached by an iterative procedure described in [4].