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# Realistic roofs without local minimum edges over a rectilinear polygon

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ABSTRACT

Computing all possible roofs over a given ground plan is a common task in automatically reconstructing a three dimensional building. In 1995, Aichholzer et al. proposed a definition of a *roof* over a simple polygon $P$ in the $xy$-plane as a terrain over $P$ whose faces are supported by planes containing edges of $P$ and making a dihedral angle $\pi/4$ with the $xy$-plane. This definition, however, allows roofs with faces isolated from the boundary of $P$ and local minimum edges inducing pools of rainwater. Very recently, Ahn et al. introduced "realistic roofs" over a rectilinear polygon with $n$ vertices by imposing two additional constraints under which no isolated faces and no local minimum vertices are allowed. Their definition is, however, restricted and excludes a number of roofs with no local minimum edges. In this paper, we propose a new definition of realistic roofs over a rectilinear polygon that corresponds to the class of roofs without isolated faces and local minimum edges. We investigate the geometric and combinatorial properties of realistic roofs and show that the maximum possible number of distinct realistic roofs over a rectilinear $n$-gon is at most $1.3211^m\binom{m}{\lfloor m/2 \rfloor}$, where $m = (n-4)/2$. We also present two algorithms that generate all realistic roofs.



## 1. Introduction

A common task in automatically reconstructing a three dimensional city model from its two dimensional map is to compute all the possible roofs over the ground plans of its buildings [4,5,11,9,10,13]. For instance, Fig. 1(a) shows a ground plan of a building in a perspective view, which is the union of two overlapping rectangles. The roof in Fig. 1(b) can be constructed by building a roof over each rectangle and taking the upper envelope of the two roofs. The roof in Fig. 1(c) can be constructed by shrinking the ground plan at a constant speed while moving it along vertically upward at a constant speed. Note that the vertical projection of the roof in Fig. 1(c) coincides with the straight skeleton of the ground plan [2,3].

For some applications, a correct or reasonable roof over a building is chosen from its set of possible roofs by considering some additional information such as its satellite images.

Aichholzer et al. [2] defined a *roof* over a simple (not necessarily rectilinear) polygon in the $xy$-plane as a terrain over the polygon such that the polygon boundary is contained in the terrain and each face of the terrain is supported by a plane containing at least one polygon edge and making a dihedral angle $\pi/4$ with the $xy$-plane. This definition, however, is not tight enough that it allows roofs with faces *isolated* from the boundary of the polygon (Fig. 2(a)) and local minimum

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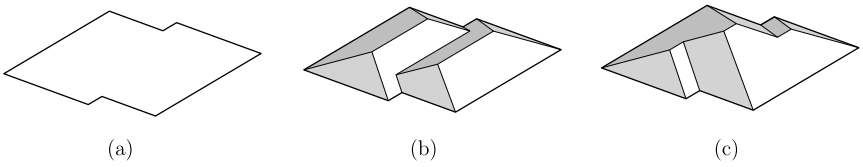


**Fig. 1.** A rectilinear ground plan and two different roofs over the plan in a perspective view.

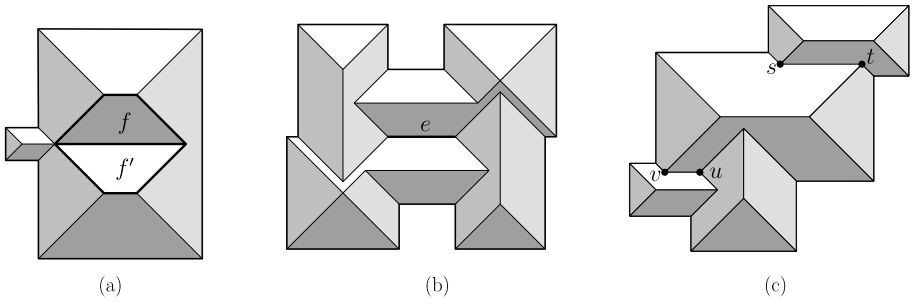


**Fig. 2.** (a) A roof with *isolated* faces $f$ and $f'$. (b) A roof with a local minimum edge $e$. (c) Not a realistic roof according to Definition 1; vertex $u$ has no adjacent vertex that is lower than itself.

edges (Fig. 2(b)) which are undesirable for some practical reasons – for example, a local minimum edge serves as a pool of rainwater, which can cause damage to the roof. Note that a pool of rainwater on a roof always contains a local minimum edge or vertex.

### 1.1. Related work

Brenner [5] designed an algorithm that computes all the possible roofs over a rectilinear polygon, but no polynomial bound on its running time is known. Recently, Ahn et al. [1] introduced "realistic roofs" over a rectilinear polygon $P$ with $n$ vertices by imposing two additional constraints to the definition of "roofs" by Aichholzer et al. [2] as follows.

**Definition 1** *([1]).* A realistic roof over a rectilinear polygon $P$ is a roof over $P$ satisfying the following constraints.

*C1*. Each face of the roof is incident to at least one edge of $P$.
*C2*. Each vertex of the roof is higher than at least one of its neighboring vertices.

They showed some geometric and combinatorial properties of realistic roofs, including a connection to the *straight skeleton* [2,3,7,6,8]. Consider a roof $R^*(P)$ over $P$ constructed by a *shrinking process*, where all of the edges of $P$ move inside, being parallel to themselves, with the same speed while moving upward along the $z$-axis with the same speed. Aichholzer et al. [2] showed that $R^*(P)$ is unique and its projection on the $xy$-plane is the straight skeleton of $P$. Ahn et al. [1] showed that $R^*(P)$ is the pointwise highest realistic roof over $P$. From the fact that $R^*(P)$ does not have a "valley", Ahn et al. [1] suggested a way of constructing another realistic roof over $P$ different to $R^*(P)$ by adding a set of "compatible valleys" to $R^*(P)$. They showed that the number of realistic roofs lies between 1 and $\binom{m}{\lfloor m/2 \rfloor}$ where $m = (n-4)/2$, and presented an output sensitive algorithm generating all combinatorial representations of realistic roofs over $P$ in $O(1)$ amortized time per roof, after $O(n^4)$ preprocessing time.

### 1.2. Our results

Constraint *C1* in Definition 1 was introduced to exclude roofs with isolated faces and constraint *C2* was introduced to avoid pools of rainwater. However, *C2* is restrictive and excludes a large number of roofs containing no local minimum edges. For example, the roof in Fig. 2(c) is not realistic according to Definition 1 though rainwater can be drained well along $uv$. Therefore, Definition 1 by Ahn et al. [1] only describes a subset of "realistic" roofs.

We introduce a new definition of realistic roofs by replacing constraint *C2* of Definition 1 with a relaxed one that excludes roofs with local minimum edges only.

**Definition 2.** A realistic roof over a rectilinear polygon $P$ is a roof over $P$ satisfying the following constraints.

*C1*. Each face of the roof is incident to at least one edge of $P$.
*C2′*. For each roof edge $uv$, $u$ or $v$ is higher than at least one of its neighboring vertices.

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