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Unifying approaches to consensus across different preference representations

Gleb Beliakov, Simon James *

Deakin University, School of Information Technology, 221 Burwood Hwy, Burwood, Victoria 3125, Australia

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ABSTRACT

Consensus measures can be useful in group decision making problems both to guide users toward more reasonable judgments and to give an overall indication of the support for the final decision. The level of consensus between decision makers can be measured in contexts where preferences over alternatives are expressed either as evaluations or scores, pairwise preferences, and weak orders, however these different representations often call for different approaches to consensus measurements. In this paper, we look at the distance metrics used to construct consensus measures in each of these settings and how consistent these are for preference profiles when they are converted from one representation to another. We develop some methods for consistent approaches across decision making settings and provide an example to help investigate differences between some of the commonly used distances.

1. Introduction

Although the fundamental problem at the heart of group decision making is how best to aggregate the preferences of individuals into a collective decision, recent research has increasingly looked to incorporate the extent to which the individuals agree, i.e. their consensus, as part of the process. Automated or semi-automated mediation can be based on how far individuals seem to deviate from the group opinion, while an overall measure of the agreement between experts can be used to accept or reject a final decision. The idea of consensus and the related ideas of majority, agreement and variation have always been important in almost all forms of decision-making, from voting in governments to the acceptance of scientific theories.

In soft-computing and decision-making, consensus can be modeled as a *fuzzy* or *soft* concept, ranging in its degree from 0 to 1 [14,32]. Such measures have been proposed in a wide range of contexts. Seminal works such as that of Bosch proposed consensus measures for sets of ranked preferences [10], while Garcia-Lapresta, Perez-Roman and their colleagues have extended such notions to weak orders [24–26,28] and other frameworks [23,27]. In these cases, individuals express their preferences over the set of alternatives or candidates by ranking them. The most commonly investigated preference structure for group-decision making, however has been for preference relations expressed over pairs of alternatives, and so a number of consensus measures have been investigated in this setting [2,14,15,33,45]. The preference for one alternative over another can be expressed in a crisp way, i.e. either alternative i is preferred to j, j is preferred to i, or the decision maker is indifferent, or alternatively as a fuzzy preference where the preference for one alternative over another takes a value in $[0, 1]$. Issues such as the rate of convergence to consensus have been investigated, as well as how the underlying distance affects the iterations required to reach consensus [18]. Consensus measures for sets of real inputs, i.e. where a score over a given scale is given to each alternative, have been explored in [4,5,13], along with generalizations such as ordinal scales [42] and qualitative assessments [1,36]. In [8], we also proposed some consensus measures for cases where individuals only vote for one candidate or alternative, which we based on ecological evenness evaluations. These different consensus settings are visually represented in Fig. 1, which could be seen as comprising a spectrum from most to least detail in terms of how much precision and detail is required from the decision makers.

In general, the consensus measures used in all settings fall into two categories: those that combine distances between each pair of decision makers, and those that look at distances between each decision maker and the overall opinion. With regard to this latter approach, we note that aggregation of preferences and individual evaluations can be performed with respect to penalty functions [3], which in turn can be interpreted as minimizing the disagreement between each of the inputs and the output.

* Corresponding author. Tel.: +61 3 9251 7481.
E-mail addresses: gleb@deakin.edu.au.au (G. Beliakov), sjames@deakin.edu.au (S. James).

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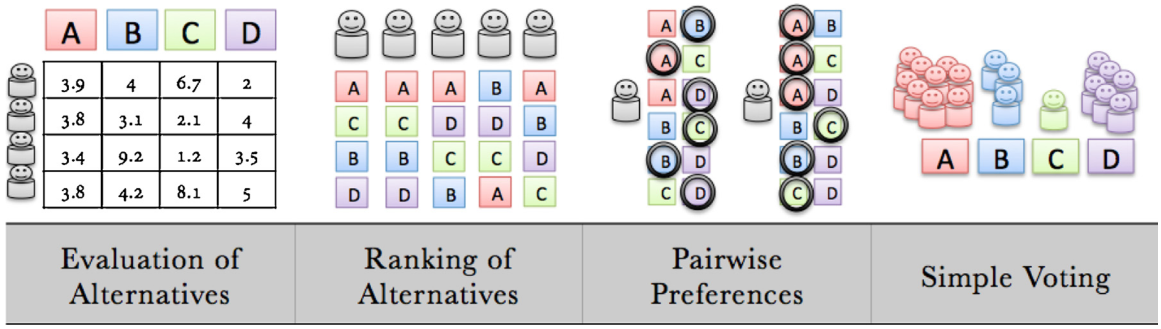

Fig. 1. Various situations with multiple alternatives/candidates and experts/voters where consensus evaluations may be useful. In each case, preferences are expressed over 4 alternatives $\{A, B, C, D\}$.

For this contribution, we focus on the way distance or dissimilarity is defined in each setting between two individuals, and whether or not consistent approaches can be taken. By consistency, we mean that if two individuals' preferences can be equivalently represented in two settings, e.g. as pairwise preferences or as weak orders, then the distance between them (and conversely, their consensus) should be independent of the setting in which they are represented. Of course, in some cases when an individual's preferences are mapped from one framework to another, information is lost, however it might still be desirable to maintain some level of comparability between different consensus evaluations. We note that methods for converting preferences from one preference structure to another have been well addressed, e.g. in [33], however to date the question of how such transformations could affect the consensus evaluations, and the resulting interpretations around agreement have received little attention. We look at some of the commonly used distances and attempt to find relationships across the preference frameworks, in the process defining some new consensus measures and evaluating the consistency of existing ones. We also look at how some of the distances employed affect some useful consensus properties such as maximum dissension and monotonicity.

The article will be set out as follows. In Section 2, we give an overview of consensus measures. In Section 3, we look in detail at some of the distances used for defining consensus measures and how these relate to each other. From these observations, in Section 4 we then propose some methods for measuring dissimilarity between sets of inputs consistently across different representations. In Section 5, we provide an example to help show the differences between some of the commonly used distances across different preference settings before making our concluding remarks in Section 6.

2. Consensus measures background

Consensus evaluations are used to give the level of agreement or overall similarity for a set of inputs. Although different models have been employed in different settings, it is usually the case that if all inputs are the same, then we should achieve a perfect level of consensus, and as more and more of the inputs differ, the level of consensus should be reduced.

2.1. Notation

Given a set of m decision-makers or voters $V = \{v_1, v_2, \ldots, v_m\}$, we consider their preferences over n candidates or alternatives $U = \{u_1, u_2, \ldots, u_n\}$. Preferences $f(v_k, u_i) \in \mathcal{P}$ can be expressed as: evaluations $x_i^k \in [0, 1]$, indicating an overall score awarded by voter k to candidate i; rankings $r_i^k \in \{1, 2, \ldots, n\}$, showing that voter k ranked candidate i as the (r_i^k)th best candidate; pairwise preferences $p_{ij}^k \in \{0, 1\}$ indicating whether or not voter k prefers candidate i to candidate j; and single votes $s_i \in \{1, 2, \ldots, n\}$ showing the proportion of voters who prefer candidate i.

There are various existing generalizations and extensions. For example, evaluations can be expressed as intervals [46], rankings can be expressed as *weak* (rather than complete) orders [28], and pairwise preferences can be expressed as either additive or multiplicative fuzzy degrees of preferences [33], i.e. $p_{ij}^k \in [0, 1]$.

2.2. Consensus properties

We define the following properties for consensus measures.

Definition 1. Let $\mathcal{C} : \mathcal{P}_{V\times U} \to [0, 1]$ denote a consensus measure where $\mathcal{P}_{V\times U}$ is a set of preference profiles pertaining to a set of voters V over a set of alternatives U, i.e. evaluations $f(v_k, u_i)$. A consensus measure is said to satisfy:

- C1 **Unanimity** when it holds that $\mathcal{C}(\mathcal{P}_{V\times U}) = 1$, if $f(v_j, u_i) = f(v_k, u_i)$ for all j, k;
- C2 **Anonymity** when it holds that $\mathcal{C}(\mathcal{P}_{V\times U}) = \mathcal{C}(\mathcal{P}_{V_\sigma \times U})$ where V_σ represents any permutation of the voters $\{v_{\sigma(1)}, v_{\sigma(2)}, \ldots, v_{\sigma(m)}\}$;
- C3 **Neutrality** when it holds that $\mathcal{C}(\mathcal{P}_{V\times U}) = \mathcal{C}(\mathcal{P}_{V\times U_\sigma})$ where U_σ represents any permutation of the alternatives $\{u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)}\}$;
- C4 **Maximum dissension** if the consensus value reaches a minimum when the voters can be partitioned into two equally sized groups $|V_1| = |V_2|$ with preferences in V_1 being as far as possible from V_2;
- C5 **Reciprocity** when the consensus value is the same if we reverse the preference ordering (or take the negation[1] of each evaluation) for each voter;
- C6 **Replication invariance** when duplicating the set of voters or inputs does not alter the level of consensus;
- C7 **Monotonicity with respect to the majority** when, if there exists a subset of unanimous voters V_{maj} comprising half of the population, any movement of the remaining voters 'closer' to the preferences expressed in V_{maj} should not decrease the level of consensus.

We could also require strict versions of unanimity (C1) and maximum dissension (C4), i.e. that a 1 or 0 can *only* be achieved for complete agreement or maximum disagreement respectively, as well as for monotonicity (C7), requiring the consensus level to increase whenever the overall disagreement decreases.

[1] For evaluations in [0, 1], a negation is a decreasing (order reversing) function $N : [0, 1] \to [0, 1]$ such that $N(0) = 1$ and $N(1) = 0$. In particular, we can consider the standard negation $N(t) = 1 - t$ which is strictly decreasing and involutive, i.e. $N(N(t)) = t$.