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Edge-independent spanning trees in augmented cubes

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ABSTRACT

Edge-independent spanning trees (EISTs) have important applications in networks such as reliable communication protocols, one-to-all broadcasting, and secure message distribution, thus their designs in several classes of networks have been widely investigated. The n-dimensional augmented cube (AQ_n) is an important variant of the n-dimensional hypercube. It is $(2n-1)$-regular, $(2n-1)$-connected ($n \neq 3$), vertex-symmetric and has diameter of $\lceil n/2 \rceil$. In this paper, by proposing an $O(N \log N)$ algorithm that constructs $2n-1$ EISTs in AQ_n, where N is the number of nodes in AQ_n, we solve the EISTs problem for this class of graphs. Since AQ_n is $(2n-1)$-regular, the result is optimal with respect to the number of EISTs constructed.

1. Introduction

A subgraph T of G is a *spanning tree* of G if T is a tree and $V(T) = V(G)$. Two spanning trees T and T' of G are *edge-independent* if T and T' are rooted at the same vertex, say r, and for each vertex v ($\neq r$) in G, the two paths from r to v, one path in each tree, share no common direct edges [10]. A set of spanning trees of G are called *edge-independent spanning trees* (EISTs as short) if they are pairwise edge-independent.

The EISTs problem is to construct a set of pairwise edge-independent spanning trees in a given graph. Because EISTs have wide applications in networks such as reliable communication protocols [1,2,14], one-to-all broadcasting [22], and secure message distribution [1,20], the EISTs problem has received much attention in recent years. For example, fault tolerance can be achieved by sending n copies of a message along n EISTs rooted at the source node. If the source node is faultless, then this scheme can tolerate up to $n-1$ faulty nodes. On the other hand, by separating a message at the source node into several parts and sending different parts of the message safely from the source node to multiple destination nodes via EISTs, every destination node can correctly obtain its own part of the message and keep the message secret from all other nodes in the transmission path.

Although the EISTs problem is very tough for arbitrary graph, several results are known for some special classes of graphs, such as planar graphs [13], even graphs [15], hypercubes [19,21,28], locally twisted cubes [6,11,16], folded hypercubes [25], crossed cubes [4,8], enhanced hypercubes [26], Möbius cubes [7,29], twisted cube [24], recursive circulant graphs [27], parity cubes [5,23] and so forth.

It is well known that hypercube is one of the most popular topologies used in parallel systems because of its simple and symmetrical structure. To improve the efficiency, many variants of hypercube have been proposed, e.g., crossed cubes, Möbius cubes, locally twisted cubes, and so on. The diameter of several variants is about half of that of the n-dimensional

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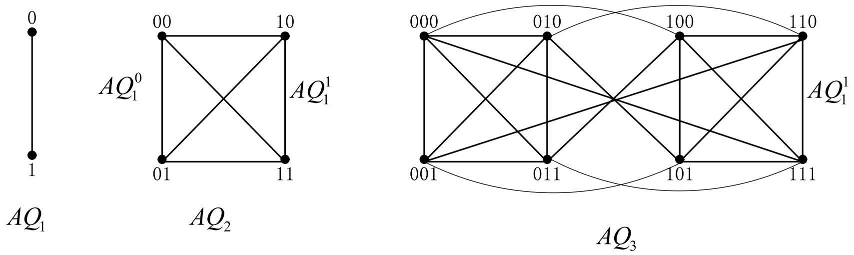

Fig. 1. Augmented cubes of dimensions 1, 2 and 3.

hypercube for a sufficient large integer n, however some of these cubes lose vertex-symmetry, making communication and computation on them vertex-specific.

As an important variant of hypercube, the augmented cube is proposed by Choudum and Sunitha in [9]. The n-dimensional augmented cube (AQ_n) is $(2n-1)$-regular, $(2n-1)$-connected ($n \neq 3$) and vertex-symmetric. AQ_n not only keeps the favorable properties of the n-dimensional hypercube (Q_n) since Q_n is a subgraph of AQ_n, but also processes some properties that Q_n does not have. For example, the diameter of AQ_n is $\lceil n/2 \rceil$ [9], almost halved Q_n's. AQ_n contains cycles of all lengths from 3 to $2n$, whereas Q_n contains only even cycles. Moreover, AQ_n is $(2n-3)$-edge-fault-tolerant pancyclic ($n \geq 2$) [17]. For $n \geq 4$, AQ_n is $(2n-3)$-fault-tolerant hamiltonian, $(2n-4)$-fault-tolerant hamiltonian connected [12]. The super vertex-connectivity of AQ_n is $4n-8$ for $n \geq 6$ and the super edge-connectivity is $4n-4$ for $n \geq 5$ [18].

In this paper, we give an $O(N \log N)$ algorithm that constructs $2n-1$ EISTs in n-dimensional augmented cubes, where N is the number of nodes in AQ_n, thereby solving the EISTs problem for this class of graphs. Since AQ_n is $(2n-1)$-regular, the number of EISTs constructed is maximum.

The rest of this paper is organized as follows. Section 2 introduces relevant notations and definitions. Then, Section 3 gives the algorithm for constructing $2n-1$ EISTs in AQ_n and shows the correctness. Finally, we give some concluding remarks in Section 4.

2. Preliminaries

Let $G = (V, E)$ be a simple graph, where $V(G)$ is the node set and $E(G)$ is the edge set. If $(x, y) \in E(G)$, we say x is adjacent to y. In this paper, we may use $x \longrightarrow y$ to denote (x, y). A neighbor of a node x is any node adjacent to x. If $V' \subseteq V(G)$, the subgraph of G induced by V' is denoted by $G[V']$. For a tree T rooted at a vertex r, we use $root(T)$ to denote r and the parent of a vertex v ($\neq r$) in T is denoted by $parent(v, T)$. We denote by $T(x, y)$ the unique path from x to y in T. A binary string u of length n is denoted by $u_{n-1}u_{n-2}\ldots u_1u_0$. For $0 \leq i \leq n-1$, we use $\overline{u_i} = 1 - u_i$ to denote the complement of u_i. A graph G_1 is isomorphic to another graph G_2 (denoted by $G_1 \cong G_2$) if and only if there exists a bijection $f : V(G_1) \to V(G_2)$ such that for any two vertices $u, v \in V(G_1)$, $(u, v) \in E(G_1)$ if and only if $(f(u), f(v)) \in E(G_2)$.

In what follows, we formally introduce the definition of augmented cubes. The n-dimensional augmented cube denoted by AQ_n can be recursively defined as below [9].

Definition 1. Let $n \geq 1$. The n-dimensional augmented cube AQ_n has 2^n vertices, each labeled by an n-bit binary string. (1) $AQ_1 = K_2$. (2) For $n \geq 2$, AQ_n is obtained by taking two copies of augmented cube AQ_{n-1}, denoted by AQ_{n-1}^0 and AQ_{n-1}^1, and adding $2 \times 2^{n-1}$ edges between the two as follows. Let $V(AQ_{n-1}^i) = \{ix | x \in V(AQ_{n-1})\}$ $(i = 0, 1)$. A vertex $u = 0u_{n-2}u_{n-3}\ldots u_0 \in V(AQ_{n-1}^0)$ is adjacent to a vertex $v = 1v_{n-2}v_{n-3}\ldots v_0 \in V(AQ_{n-1}^1)$ if and only if for every $0 \leq i \leq n-2$, one of the following two conditions holds: (a) $u_i = v_i$. In this case, the edge (u,v) is called a hypercube edge; (b) $u_i = \overline{v_i}$. In this case, the edge (u,v) is called a complement edge.

The augmented cubes AQ_1, AQ_2 and AQ_3 are demonstrated in Fig. 1.

To expatiate the constructing rule of EISTs, we need the following notations. We first give the definition of n-dimensional adjacent vertex in AQ_n.

Definition 2. For any integer $n \geq 1$ and $u = u_{n-1}u_{n-2}\ldots u_1u_0 \in V(AQ_n)$, the k-dimensional adjacent vertex of u in AQ_n $(0 \leq k \leq n-1)$ is defined as follows. (1) If $k = 0$, then the 0-dimensional adjacent vertex of u in AQ_n (denoted by $N_0(u)$) is $u_{n-1}u_{n-2}\ldots u_1\overline{u_0}$. (2) If $k \geq 1$, then there are two k-dimensional adjacent vertices of u in AQ_n. One is denoted by $N_k^h(u) = u_{n-1}u_{n-2}\ldots\overline{u_k}\ldots u_1u_0$; the other one is denoted by $N_k^c(u) = u_{n-1}u_{n-2}\ldots\overline{u_k \ldots u_1u_0}$. Furthermore, we use N_k^* to denote N_k^h or N_k^c. Moreover, we define $N_k^*(S) = \{N_k^*(x) | x \in S\}$, where S is a subset of $V(AQ_n)$.

Let G be a subgraph of AQ_k^0. For any vertices $u, v \in V(G)$ and $(u, v) \in E(G)$, since $AQ_k^0 \cong AQ_k^1$, $N_k^h(u)$ must be adjacent to $N_k^h(v)$. Then we define $N_k^h(G)$ as the corresponding subgraph of G in AQ_k^1, where $V(N_k^h(G)) = \{N_k^h(x) | x \in V(G)\}$ and $E(N_k^h(G)) = \{(N_k^h(u), N_k^h(v)) | (u, v) \in E(G)\}$. It is easy to prove the following lemma.