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# Estimating the subsystem reliability of bubblesort networks

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ABSTRACT

The exact reliability of a complicated network system is usually difficult to determine, and numerical approximations may play a crucial role in indicating the reliable probability that a system is still operational under a specified suite of conditions. In this paper, we establish upper and lower bounds on the first-order subsystem reliability of bubblesort networks using the probabilistic fault model. Numerical results show that the curves of upper- and lower-bounded reliability are in good agreement, especially when the node reliability is at a low level.



## 1. Introduction

As the size of a network system grows, the number of potential node failures may increase rapidly [10,20]. It is of great importance to quantify the effect of node failures so that a fault-tolerant design can be achieved [8,21]. Reliability has long been considered as an index of evaluating the degree of fault tolerance for a complicated system. In general, a system's reliability is defined as the probability that the system has survived during the time interval $[0, t]$, supposed that it was operational at time $t = 0$. Traditional measures of reliability include the terminal reliability [14] and the distance reliability [19]. Another belongs to the task-based dependability, which is defined as the probability that a connected group of working nodes is available for task execution [6].

Reliability evaluation of hypercube and mesh networks has been well studied. Analytic models developed to analyze the hypercube's reliability address both node- and link-failure schemes [3,14]. Zhu et al. [18] analyzed the reliability of the folded hypercube; Chen et al. [5] developed a technique for deriving lower bounds on the connectivity probability for both 2-D and 3-D meshes. Later, Wang et al. [15] investigated fault tolerance analysis of mesh networks with uniform versus nonuniform node failure distribution, and Liang et al. [12] studied the connection probability for 2-D meshes and tori.

In particular, Chang and Bhuyan [3] first formalized the probabilistic fault model for computing the subcube reliability of hypercubes. Later, Wu and Latifi [20] applied the probabilistic fault model to estimate the substar reliability based on star graphs. Recently, Lin et al. [13] studied the subgraph reliability of the arrangement graph, and Li et al. [11] developed a combinatorial formulation for computing both upper and lower bounds on the subsystem reliability of $(n, k)$-star graph. In this paper, we are going to establish analytic bounds on the first-order subsystem reliability of bubblesort networks using

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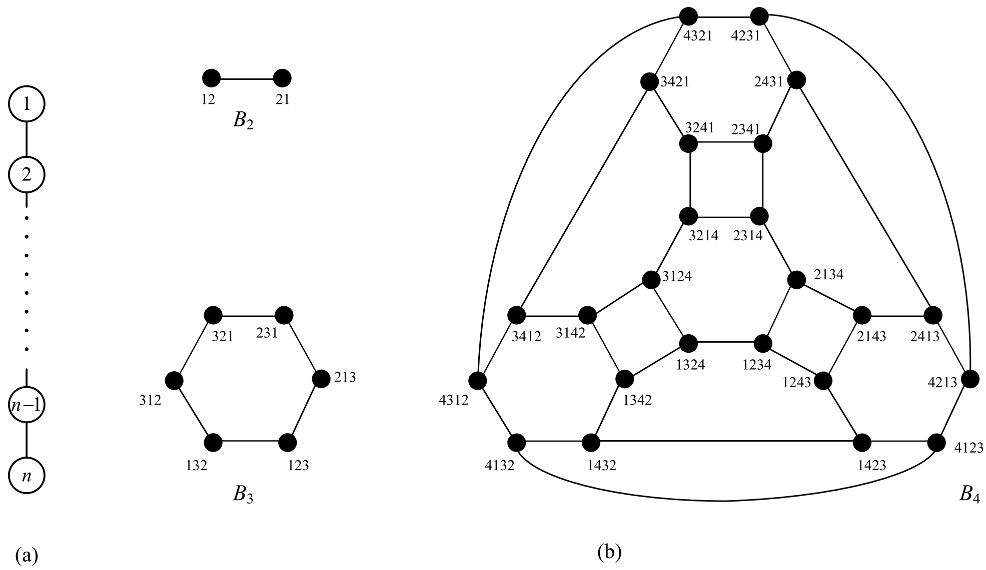


**Fig. 1.** (a) The generator graph of $B_n$; (b) $B_2$, $B_3$, and $B_4$.

the probabilistic fault model. The bubblesort network emerged on the basis of the well-known bubblesort algorithm, and it possesses many attractive topological properties, including node-symmetry, regularity, scalability, small diameter, and so on [1,2,7,9,16,17]. The presented numerical results exhibit that the curves of upper- and lower-bounded subsystem reliability of the bubblesort network are in good agreement, and this also reveals that our estimates are close to the true values.

The remainder of this paper is structured as follows. Section 2 introduces the fundamentals of the probabilistic fault model and topological properties of the bubblesort network. Sections 3 and 4 present lower and upper bounds on the subsystem reliability of bubblesort networks, respectively. Section 5 shows numerical results to validate the established formulation. Finally, our concluding remarks are drawn in Section 6.

## 2. Preliminaries

The underlying topology of a network system is modeled as a graph. Throughout this paper graphs are finite, simple, and unless specified otherwise, undirected. We follow the standard graph-theory terminology given in [4].

### 2.1. Bubblesort networks

The $n$-bubblesort network [1], $B_n$, $n \geq 2$, is an undirected graph consisting of $n!$ nodes, each of which corresponds to exactly one permutation of $n$ distinct symbols $x_1, x_2, \cdots, x_n$. Without loss of generality, we consider $\langle n \rangle \triangleq \{1, 2, \ldots, n\}$ as the associated set of $n$ distinct symbols. Two nodes are joined by a $j$-link if and only if the permutation of one node can be obtained from the other by swapping the $(j-1)$th digit and the $j$th digit, where $2 \leq j \leq n$. Thus, $B_n$ is a Cayley graph completely specified by a set of $n-1$ generators [1], each of which is a transposition of adjacent symbols. Fig. 1(a) depicts the generator graph of $B_n$, which happens to be an $n$-path, a path of length $n$. Fig. 1(b) illustrates $B_2$, $B_3$, and $B_4$.

The $B_n$ can be decomposed of $n$ disjoint $(n-1)$-bubblesort networks. Apparently, the generator graph of $B_{n-1}$ is an $(n-1)$-path, and the generator graph of $B_n$ contains only two subgraphs that are isomorphic to an $(n-1)$-path: one is induced by $\{1, 2, \ldots, n-1\}$, and the other is induced by $\{2, \ldots, n-1, n\}$. The two $(n-1)$-paths can be produced via the removal of link $\{n-1, n\}$ or $\{1, 2\}$ from the generator graph of $B_n$, respectively. Therefore, there are exactly two ways to partition $B_n$ into $n$ disjoint $B_{n-1}$-subgraphs for $n \geq 3$. Let $L_j$ denote the set of all $j$-links in $B_n$. Then, both $B_n - L_2$ and $B_n - L_n$ consists of $n$ disjoint $B_{n-1}$-subgraphs (see Fig. 2); thus, there are $2n$ distinct $B_{n-1}$-subgraphs in $B_n$. The decomposition of $B_n$ can be obtained in a different way: For two integers $i \in \{1, n\}$ and $x \in \langle n \rangle$, let $V_n^{i:x}$ be the set of all nodes whose $i$th digit is identical to $x$ in $B_n$, and let $B_n^{i:x}$ denote the subgraph of $B_n$ induced by $V_n^{i:x}$. Accordingly, $B_n^{i:x}$ is isomorphic to $B_{n-1}$.

### 2.2. Probabilistic fault model

Chang and Bhuyan [3] introduced the probabilistic fault model to evaluate the subsystem reliability of a multiprocessor system. In this model, every node has a homogeneous node reliability, and node failures appear independently. For $0 \leq k \leq n-2$, let $R_n^{(k)}(p)$ denote the $k$th-order subsystem reliability of $B_n$ – the probability that there exists at least one fault-free