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Network Movement Games [☆,☆☆]

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ABSTRACT

We introduce a new class of games, called *Network Movement Games*, naturally related to the movement problems of [7], which models the spontaneous creation of multi-hop communication networks by the distributed and uncoordinated movement of k selfish mobile devices placed on the nodes of an underlying graph. Devices are players aiming to find the final positions which achieve a global property of the induced subnetwork. We actually focus on solutions (i.e. Nash equilibria) connecting all the players while minimizing the distances from their home locations. We show that the game always admits a pure Nash equilibrium, and that the convergence after a finite number of improving moves is guaranteed only when players perform their best possible moves; in this case, we provide tight bounds on the speed of convergence to Nash equilibria, both when initial positions are arbitrary and when players start at their home positions. As for the Nash equilibria performances, we first prove that the price of stability is 1 (i.e., an optimal solution is also a Nash equilibrium). Furthermore, we provide tight bounds on the price of anarchy, also showing that better performances are obtained when players start at their home positions. Finally, through extensive experiments, we show that high bounds on the price of anarchy as well as high convergence time of best move dynamics to Nash equilibria are only due to pathological worst cases. Moreover, quite often improving move dynamics (i.e., where players do not necessarily perform a best possible move) converge to Nash equilibria on the tested instances even though the class of instances does not guarantee the convergence.

1. Introduction

Global communication and service infrastructures like the Internet are characterized by decentralization, autonomy, and general lack of coordination among the heterogeneous network entities, the network in turn intrinsically being a common playground for a large number of users. In such a highly elusive and mutable setting, the general mismatch between the network optimization goals and the competing users private interests motivated an extensive research on (algorithmic)

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game-theoretical frameworks aiming to characterize the system outcome by suitable stable solution concepts like the Nash equilibrium [22].

In this paper we investigate the movement problems of [7], where the authors study under a classical centralized setting the problem of planning the coordinated motion of a collection of devices placed on the vertices of an underlying graph so that their final positions achieve a global property of the network while minimizing the maximum or average movement. Several movement problems fit into this general framework. A relevant variation is the so called ConSum problem, in which the aim is that of achieving connectivity while minimizing the total movement. We consider a non-cooperative scenario in which the mobile users/devices are autonomous and we introduce a new class of games, called *Network Movement Games*; this class of games is actually the game-theoretical version of the ConSum problem of [7], where the objective is to minimize the total movement of coordinated devices to reach connectivity.

More precisely, we consider a collection $\{1, \ldots, k\}$ of selfish mobile devices (or players) and a given undirected graph $G = (V, E)$. Each device i is initially placed on a home location $h_i \in V$. Each player of the game has a set of pure strategies, i.e., a finite set of possible actions. A pure strategy profile for the players is a tuple containing a pure strategy for each player. Given any strategy profile $S = (s_1, \ldots, s_k)$, where $s_i \in V$ denotes the node (i.e., the strategy) selected by the device i, a player i is “directly” connected to all the devices j at distance (i.e., the length of a shortest path connecting s_i and s_j in G) at most a given threshold r, while she can communicate to the remaining ones in a multi-hop fashion. The graph G and the locations selected by the devices are common knowledge among the players.

A strategy profile is a pure Nash equilibrium if no player can benefit by changing her pure strategy unilaterally, i.e. while the other players keep their strategies unchanged. We only study pure Nash equilibria so we often omit the word “pure”. In this paper we are interested in solutions (i.e. Nash equilibria) connecting all the devices, that is we consider the goal of achieving the connectivity. Namely, each player chooses her final position in order to increase connectivity while minimizing her own distance from the home location.

Each player has then two goals: to minimize the number of players she cannot reach via multi-hop communication (connection cost) and to minimize the distance from her home location (distance cost). We enforce that the goal of the players is firstly to achieve connectivity and only secondly to care about minimizing the distance to their home locations by restricting our attention to a suitable cost function defined as the distance cost plus $D + 1$ times the connection cost, with D being the diameter of G. Each device is interested in selecting a position (strategy) minimizing her own cost. The social cost of a solution is simply the sum of all players’ costs. Notice that the final social cost in a solution where all the players are connected, coincides with the sum of the distances of the players from their home locations.

One of the main tools for evaluating the degradation of the system performance induced by the lack of coordination of selfish players is the price of anarchy (PoA) [19,23], a measure that compares the social cost of the worst case Nash equilibrium to the social optimum one. A related optimistic measure for evaluating the cost of the best possible Nash equilibrium is the price of stability (PoS) [4], a measure that compares the social cost of the best Nash equilibrium to the social optimum one. The complexity of computing a Nash equilibrium and the speed of convergence to such solutions have been extensively studied for many classes of non-cooperative games, like in [1,12] in the context of congestion games.

Our games can be also regarded as a variation on the well known Network Creation Games [3,8,9,13,20,25,26], extensively investigated in the literature to capture scenarios in which different autonomous network entities with local optimization goals concur in the creation of the network infrastructure required to satisfy their communication needs. In a Network Creation Game every player i is identified with a node of a given graph and has to decide what edges incident to i the player creates (or buys, or builds). The edges that the players create form a graph which is the result of the game. Thus each player has two goals: to minimize the cost spent buying a set of adjacent edges and to minimize the average or maximum distance to all other nodes. When the pattern of the game is to get all connected, the players attempt to build an efficient network that interconnects everyone by creating a set of adjacent edges, rather than by selecting a set of nodes in the graph.

It is worth emphasizing that our model can be applied to a wide variety of contexts. For instance, a concrete scenario of our model could be the following: natural disasters or calamity are unforeseeable events which cannot be prevented and therefore rescuers need to organize themselves in loco. Each rescuer (or rescue team) has a specific operation area to work on (for instance firefighters are in the fire zone, doctors are in the zone with wounded, and so on) representing his home position. A reliever is equipped with a radio able to transmit and receive data from/to radios at distance at most r. Multi-hop connections are allowed. On the one hand rescuers require to build up a communication network in order to coordinate themselves and to exchange information; on the other hand they want (and indeed have) to be as close as possible to their own home positions. Furthermore, consider a different scenario in which autonomous robots, each one monitoring a specific area (i.e., the home position), with limited wireless connectivity and limited mobility in the field because of energy and resource constraints, wish to minimize the use of the resources to form a reliable radio network (see for example [6]). Note that in the above described scenarios we can make different assumptions: the game may be played “on the field”, and in this case each player has to know the locations through GPS or other range-based localization techniques, or the game can be played “in the chair”, meaning that it is a pre-processing phase where agents negotiate for positions to stay and, once reached a Nash equilibrium, they move to the corresponding final positions.