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Local testability from words to traces, a suitable definition

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ABSTRACT

The family of locally testable languages has been extensively studied. We propose an extension of the notion of local testability from the free monoid to the free partially commutative monoid (the so called trace monoid). We show that to formalize the notion of locality in traces is conceptually difficult, and we introduce a new kind of factor which takes into account these difficulties. Thus we define a locally testable trace language as a union of classes of a new equivalence relation of a finite index, which is proved to be a congruence using some combinatorics on traces. Then we give a set-theoretic characterization of the family of locally testable trace languages, in terms of some sets called here quasi-ideals, as a generalization of ideals in words. Finally, analyzing the extreme cases of the free monoid and the free commutative monoid, we prove that this new family becomes exactly that of locally testables languages in the case of words.
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1. Introduction

A fundamental class in the theory of formal languages is that of star-free languages. It appears in many different approaches to automata theory, and has been extensively investigated. A language is star-free if it can be obtained by applying Boolean operations and concatenation product to the letters of the alphabet. A well-known result of Schützenberger [29] established in 1965 that a language is star-free if and only if its syntactic monoid is finite and have only trivial subgroups. This marked a milestone from which the theory of varieties was initiated, as exposed by Eilenberg [13]. We refer the reader to [24] for a more recent overview.

An important and also very studied subfamily of star-free languages is that of locally testable languages, introduced in [21,13]. A language is locally testable if membership of a word is determined by "local" properties of the word, actually by its prefixes, factors and suffixes up to some length that depends on the language. Intuitively, two words u and v are equivalent under $\sim_k$ if they can not be distinguished by a finite automaton equipped with a window of size k to scan the word. We assume that the window can also be moved beyond the first and the last letter of the word so that the prefixes and the suffixes of length $< k$ can be read, and the results of the scanning are recordered, without regard to the order in which they appear or their multiplicity. Thus a complete scan of a word u indicates exactly what are the factors of u of length k and what are the prefixes and the suffixes of length $< k$.

Locally testable languages are characterized by a deep algebraic property of their syntactic semigroup, stated independently by Brzozowski and Simon in [6] and by McNaughton in [20]. We also cite [30] for an independent characterization using semigroups. The first polynomial time algorithm to decide if the language recognized by a deterministic finite automaton is locally testable is given in [18]. Locally testable languages have various generalizations (see for example [3]) and

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have been the subject of a lot of papers. For applications, they are used, for example, in the study of DNA and informational macromolecules in biology.

Our aim in this paper is to generalize the notion of local testability to trace languages.[1]

The study of the theory of trace languages was initiated by A. Mazurkiewicz in 1977 [19], and was later developed, motivated by its interpretation as a model to describe the behaviour of concurrent systems. Actually, Cartier and Foata had started in 1969 [7] to work on commutations, in order to solve some problems of rearrangements of words. When an independence relation I is fixed over the letters of an alphabet, stating what letters can commute, a trace is identified with an equivalence class of words that can be obtained from one another by switching successively some consecutive independent letters. A trace language is a set of traces. In 1981 Bertoni et al. [4] firstly presented the connection between traces languages and the subsets of free partially commutative monoids. Thereafter the theory of trace languages was investigated as a generalization of the theory of languages in free monoids. The reader is referred to [1,10] for surveys, and to the book [11] for a collection of chapters on different topics about trace theory, written by different authors, and for further references on traces besides those mentioned along this paper.

The generalization of "local" properties to traces is conceptually difficult and it seems to be a contradiction, since in a trace a letter can move all over its length.[2]

We would like to define a locally testable trace language as a union of classes of some congruence of a finite index, as it is made for words, which immediately yields recognizability. In the free monoid the equivalence $\sim_k$ is trivially a congruence. But, we will show that to formalize the notion of locality and to find a congruence on traces, which preserves the equalities of prefixes, factors and suffixes, is not easy. The crucial point is that the equality of the factors of two traces has no reference to the contexts in which they appear. So we introduce an equivalence relation which takes into account the contexts in which the factors of a trace occur, and the commutation properties of these contexts with respect to the letters of the alphabet. A new kind of factor is defined, depending on the letters that 'commute' (actually absolutely commute) with its left context and with its right context in a trace. These factors are said left-reachable and right-reachable by some letters. To prove that this equivalence is a congruence is not trivial, and involves some combinatorics on traces. We can now define a subfamily of recognizable trace languages that we call locally testable, considering unions of classes under this congruence, which has a finite index.

Then we introduce the notion of quasi-ideal, as a generalization of principal ideals in the free monoid. This leads us to state a set-theoretic characterization of locally testable trace languages, as Boolean combinations of quasi-ideals, in the same way that locally testable languages are Boolean combinations of ideals. Finally we analyze what happens in the extreme cases of the free monoid and the free commutative monoid and it is interesting to see that we fall on the classical result for words. Indeed in the free monoid our family of locally testable trace languages coincides with that of locally testable languages. For all these reasons we think that our definition is a suitable, or at least interesting, proposition for local testability in trace monoids. Of course it encourages further work in order to find a decision algorithm for the membership of a trace language to this family.

The paper is organized as follows. In Section 2 we recall a few preliminaries and notation about trace monoids and about locally testable languages. In Section 3 we describe the process that led us to the choice of a suitable equivalence relation. In Section 4 we introduce our equivalence relation, we prove that it is a congruence, and we define the family of locally testable trace languages. In Section 5 we introduce the notion of quasi-ideal, we give a set-theoretic characterization of this family in terms of quasi-ideals, and we investigate some consequences. In the last section we consider the two extreme cases of trace monoids.

A preliminary version, without proofs, of some results contained in this paper appeared in [15].

2. Preliminaries and known results

Let us recall some definitions and known results that we shall need in the next sections.

2.1. Trace monoids

In this paper A is a finite alphabet. Then A^* is the free monoid and A^+ is the free semigroup generated by A. Let $I \subseteq A \times A$ be a symmetric and irreflexive relation called the *independence relation* or *commutation relation*. Though the relation I is meant to define the pairs of letters that are allowed to commute, for technical reasons it is always considered that a letter does not commute with itself, hence the irreflexivity of I. We consider the congruence $\sim_I$ of A^* generated by the set of pairs (ab, ba) with $(a, b) \in I$. The quotient of A^* by this congruence is called the *free partially commutative monoid* (or *trace monoid*) generated by A with respect to I and it is denoted by $M(A, I)$. The empty word (trace) is denoted by 1.

We denote $M(A, I) \setminus \{1\}$ by $M^+(A, I)$. We shall sometimes omit A and I, and simply denote $M(A, I)$ by M and $M^+(A, I)$ by M^+.

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[1] This work was initiated many years ago when the author was studying in Palermo, supervised by Antonio Restivo, to whom this special issue is dedicated.

[2] It is as if we wanted to catch the smoke ("acchiappare il fumo"), said Antonio.

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