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Approximability Issues for Unconstrained and Constrained Maximization of Half-Product Related Functions

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Abstract

We address the Boolean programming problem of maximizing a half-product function, with and without a linear knapsack constraint. Maximizing the half-product can be done in polynomial time. Adding a knapsack constraint makes the problem non-approximable within a constant factor, provided the coefficients in the linear part of the function are negative. For maximizing a function with positive coefficients in the linear part we develop a fully polynomial-time approximation scheme.

Keywords: Half-product, quadratic knapsack, non-approximability, FPTAS

1. Introduction

The half-product function is a special form of a (pseudo) Boolean quadratic function that has been studied since the 1990s; see [2] where this function and the term "half-product" were introduced. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be a vector with n Boolean components. Consider the function

$$h(\mathbf{x}) = \sum_{1 \le i < j \le n} \alpha_i \beta_j x_i x_j - \sum_{j=1}^{n} \gamma_j x_j, \tag{1}$$

where for each j, $1 \le j \le n$, the coefficients $\alpha_j$and β_j are non-negative integers, while γ_j are integers which may be of any sign.

Partly, the interest in the problems of *minimizing* the functions related to the half-product is due to their applications to scheduling problems with min-sum objective functions. Such problems include the single machine problems of minimizing the weighted total earliness and tardiness about a common due date, of minimizing the weighted total

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