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Structural operational semantics for non-deterministic processes with quantitative aspects

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ABSTRACT

Recently, unifying theories for processes combining non-determinism with quantitative aspects (such as probabilistic or stochastically timed executions) have been proposed with the aim of providing general results and tools. This paper provides two contributions in this respect. First, we present a general GSOS specification format and a corresponding notion of bisimulation for non-deterministic processes with quantitative aspects. These specifications define labelled transition systems according to the ULTraS model, an extension of the usual LTSs where the transition relation associates any source state and transition label with *state reachability weight functions* (like, e.g., probability distributions). This format, hence called *Weight Function GSOS* (WF-GSOS), covers many known systems and their bisimulations (e.g. PEPA, TIPP, PCSP) and GSOS formats (e.g. GSOS, Weighted GSOS, Segala-GSOS).

The second contribution is a characterization of these systems as coalgebras of a class of functors, parametric in the weight structure. This result allows us to prove *soundness* and *completeness* of the WF-GSOS specification format, and that bisimilarities induced by these specifications are always congruences.

1. Introduction

Process calculi and labelled transition systems have proved very successful for modelling and analysing concurrent, non-deterministic systems. This success has led to many extensions dealing with quantitative aspects, whose transition relations are endowed with further information like probability rates or stochastic rates; see [5,6,15,20,25] among others. These calculi are very effective in modelling and analysing quantitative aspects, like performance analysis of computer networks, model checking of time-critical systems, simulation of biological systems, probabilistic analysis of security and safety properties, etc.

Each of these calculi is tailored to a specific quantitative aspect and for each of them we have to develop a quite complex theory almost from scratch. This is a daunting and error-prone task, as it embraces the definition of syntax, semantics, transition rules, various behavioural equivalences, logics, proof systems; the proof of important properties like congruence of behavioural equivalences; the development of algorithms and tools for simulations, model checking, etc. This situation would naturally benefit from general *frameworks* for LTS with quantitative aspects, i.e., mathematical *meta*models offering general methodologies, results, and tools, which can be uniformly instantiated to a wide range of specific calculi and models.

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In recent years, some of these theories have been proposed; we mention *Segala systems* [27], *Functional Transition Systems* (FuTS) [22], *weighted labelled transition systems* (WLTSs) [13,20], and *Uniform Labelled Transition Systems* (ULTraS), introduced by Bernardo, De Nicola and Loreti specifically as "a uniform setting for modelling non-deterministic, probabilistic, stochastic or mixed processes and their behavioural equivalences" [5].

A common feature of most of these meta-models is that their labelled transition relations do not yield simple states (e.g., processes), but some mathematical object representing quantitative information about "how" each state can be reached. In particular, transitions in ULTraS systems have the form $P \xrightarrow{a}\!\!\rhd \rho$ where ρ is a *state reachability weight function*, i.e., a function assigning a *weight* to each possible state.[2] By suitably choosing the set of weights, and how these functions can be combined, we can recover ordinary non-deterministic LTSs, probabilistic transition systems, stochastic transition systems, etc. As convincingly argued in [5], the use of weight functions in place of plain processes simplifies the combination of non-determinism with quantitative aspects, like in the case of EMPA or PEPA. Moreover, it paves the way for general definitions and results, an important example being the notion of $\mathcal{M}$-bisimulation [5].

Albeit quite effective, these meta-models are at their dawn, with many results and techniques still to be developed. An important example of these missing notions is a *specification format*, like the well-known GSOS, ntyft/ntyxt and ntree formats for non-deterministic labelled transition systems. These formats are very useful in practice, because they can be used for ensuring important properties of the system; in particular, the bisimulations induced by systems in these formats is guaranteed to be a congruence (which is crucial for compositional reasoning). From a more foundational point of view, these frameworks would benefit from a categorical characterization in the theory of coalgebras and bialgebras: this would allow a cross-fertilizing exchange of definitions, notions and techniques with similar contexts and theories.

In this paper, we provide two main contributions in this respect. First, we present a GSOS-style format, called *Weight Function GSOS* (WF-GSOS), for the specifications of non-deterministic systems with quantitative aspects. The judgement derived by rules in this style is of the form $P \xrightarrow{a}\!\!\rhd \psi$, where P is a process and ψ is a *weight function term*. These terms describe weight functions by means of an *interpretation*; hence, a specification given in this format defines a ULTraS. By choosing the set of weights, the language of weight function terms and their interpretation, we can readily capture many quantitative notions (probabilistic, stochastic, etc.), and different kinds of non-deterministic interactions, covering models like PEPA, TIPP, PCSP, EMPA, among others. Moreover, the WF-GSOS format supports a general definition of *(strong) bisimulation*, which can be readily instantiated to the various specific systems.

The second contribution is more fundamental. We provide a general categorical presentation of these non-deterministic systems with quantitative aspects. Namely, we prove that ULTraS systems are in one-to-one correspondence with coalgebras of a precise class of functors, parametric on the underlying weight structure. Using this characterization we define the abstract notion of *WF-GSOS distributive law* (i.e. a natural transformation of a specific shape) for these functors. We show that each WF-GSOS specification yields such a distributive law (i.e., the format is sound); taking advantage of Turi–Plotkin's bialgebraic framework, this implies that the bisimulation induced by a WF-GSOS is always a congruence, thus allowing for compositional reasoning in quantitative settings. Additionally, we extend the results we presented in [24] proving that the WF-GSOS format is also *complete*: every abstract WF-GSOS distributive law for ULTraSs can be described by means of some WF-GSOS specification.

The rest of the paper is structured as follows. In Section 2 we recall Uniform Labelled Transition Systems, and their bisimulation. In Section 3 we introduce the *Weight Function GSOS* specification format for the syntactic presentation of ULTraSs. In Section 4 we provide some application examples, such as a WF-GSOS specification for PEPA and the translations of Segala-GSOS and WGSOS specifications in the WF-GSOS format. The categorical presentation of ULTraSs and WF-GSOS, with the results that the format is sound and complete and bisimilarity is a congruence, are in Section 5. Final remarks, comparison with related work and directions for future work are in Section 6.

2. Uniform Labelled Transition Systems and their bisimulation

In this section we recall and elaborate the definition of ULTraSs, and define the corresponding notion of (coalgebraically derived) bisimulation; finally we compare it with the notion of $\mathcal{M}$-bisimulation presented in [5]. Although we focus on the ULTraS framework, the results and methodologies described in this paper can be ported to similar formats (like FuTS [22]), and more generally to a wide range of systems combining computational aspects in different ways.

2.1. Uniform Labelled Transition Systems

ULTraSs are (non-deterministic) labelled transition systems whose transitions lead to *state reachability weight functions*, i.e. functions representing quantitative information about "how" each state can be reached. Examples of weight functions include probability distributions, resource consumption levels, or stochastic rates. In this light, ULTraSs can be thought of as

[2] The reader aware of advanced process calculi will be not baffled by the fact that targets are not processes. Well known previous examples are the LTS abstractions/concretions for π-calculus, for the applied π-calculus, for the ambient calculus, etc.

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