

# Accepted Manuscript

Upper bounds on fourier entropy

Sourav Chakraborty, Raghav Kulkarni, Satyanarayana V. Lokam, Nitin Saurabh

PII: S0304-3975(16)30119-0  
DOI: <http://dx.doi.org/10.1016/j.tcs.2016.05.006>  
Reference: TCS 10755

To appear in: *Theoretical Computer Science*

Received date: 31 October 2015  
Revised date: 17 April 2016  
Accepted date: 2 May 2016

Please cite this article in press as: S. Chakraborty et al., Upper bounds on fourier entropy, *Theoret. Comput. Sci.* (2016), <http://dx.doi.org/10.1016/j.tcs.2016.05.006>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Upper Bounds on Fourier Entropy

Sourav Chakraborty<sup>a</sup>, Raghav Kulkarni<sup>b</sup>, Satyanarayana V. Lokam<sup>c</sup>, Nitin Saurabh<sup>d</sup>

<sup>a</sup>Chennai Mathematical Institute, Chennai, India. [sourav@cmi.ac.in](mailto:sourav@cmi.ac.in)

<sup>b</sup>Centre for Quantum Technologies, Singapore. [kulraghav@gmail.com](mailto:kulraghav@gmail.com)

<sup>c</sup>Microsoft Research India, Bangalore, India. [satya@microsoft.com](mailto:satya@microsoft.com)

<sup>d</sup>The Institute of Mathematical Sciences, Chennai, India. [nitin@imsc.res.in](mailto:nitin@imsc.res.in)

---

## Abstract

Given a function  $f : \{0, 1\}^n \rightarrow \{+1, -1\}$ , its *Fourier Entropy* is defined to be  $-\sum_S \hat{f}(S)^2 \log \hat{f}(S)^2$ , where  $\hat{f}$  denotes the Fourier transform of  $f$ . In the analysis of Boolean functions, an outstanding open question is a conjecture of Friedgut and Kalai (1996), called the Fourier Entropy Influence (FEI) Conjecture, asserting that the Fourier Entropy of any Boolean function  $f$  is bounded above, up to a constant factor, by the total influence (= average sensitivity) of  $f$ .

In this paper we give several upper bounds on the Fourier Entropy. We first give upper bounds on the Fourier Entropy of Boolean functions in terms of several complexity measures that are known to be bigger than the influence. These complexity measures include, among others, the logarithm of the number of leaves and the average depth of a parity decision tree. We then show that for the class of Linear Threshold Functions (LTF), the Fourier Entropy is  $O(\sqrt{n})$ . It is known that the average sensitivity for the class of LTF is  $\Theta(\sqrt{n})$ . We also establish a bound of  $O_d(n^{1-\frac{1}{4d+6}})$  for general degree- $d$  polynomial threshold functions. Our proof is based on a new upper bound on the *derivative of noise sensitivity*. Next we proceed to show that the FEI Conjecture holds for read-once formulas that use AND, OR, XOR, and NOT gates. The last result is independent of a result due to O'Donnell and Tan [1] for read-once formulas with *arbitrary* gates of bounded fan-in, but our proof is completely elementary and very different from theirs. Finally, we give a general bound involving the first and second moments of sensitivities of a function (average sensitivity being the first moment), which holds for real-valued functions as well.

Download English Version:

<https://daneshyari.com/en/article/4952425>

Download Persian Version:

<https://daneshyari.com/article/4952425>

[Daneshyari.com](https://daneshyari.com)