Applied Soft Computing 29 (2015) 256–269

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making

Decui Liang [a,*], Witold Pedrycz [b,c,e], Dun Liu [d], Pei Hu [d]

[a] School of Management and Economics, University of Electronic Science and Technology of China, Chengdu 610054, China
[b] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2G7
[c] System Research Institute, Polish Academy of Sciences, Warsaw, Poland
[d] School of Economics and Management, Southwest Jiaotong University, Chengdu 610031, China
[e] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 19 March 2013
Received in revised form
10 September 2014
Accepted 7 January 2015
Available online 14 January 2015

Keywords:
Linguistic terms
Group decision making
Loss function
Decision-theoretic rough sets
Three-way decisions

ABSTRACT

Based on decision-theoretic rough set model of three-way decisions, we augment the existing model by introducing linguistic terms. Considering the two types of parameters being used in the three-way decisions with linguistic assessment, a certain type of novel three-way decisions based on the Bayesian decision procedure is constructed. In this way, three-way decisions with decision-theoretic rough sets are extended to the qualitative environment. With the aid of multi-attribute group decision making, the values of these parameters are determined. An adaptive algorithm supporting consistency improvement of multi-attribute group decision making is designed. Then, we optimize the scales of the linguistic terms with the use of particle swarm optimization. The values of these parameters of three-way decisions are aggregated when proceeding with group decision making. Finally, the proposed model of three-way decisions with linguistic assessment is applied to the selection process of new product ideas.

1. Introduction

Three-way decisions, consisting of acceptance, non-commitment (or further investigation) and rejection, are commonly encountered in problem solving strategies occurring in many decision process [56]. This way of decision-making has been applied to many domains, such as environmental precaution [8], text classification [15], information filtering [16], risk decision [17], cluster [20,57], investment decision [21], government decision [24], shadowed sets [32], email filtering [63], etc. Pawlak's rough sets [29,55] provide a certain convenient theoretical interpretation of the three-way decisions. The lower approximation and the upper approximation of rough sets [29,50] divide the universe of discourse (space) into the three pairwise disjoint regions: positive region, boundary region and negative region. The notion of three-way decisions based on the three pairwise disjoint regions was proposed by Yao [51,53,54]. The three-way decisions comprise positive rules, boundary rules and negative rules. The positive rules associated with the positive region produce acceptance decisions. The negative rules coming with the negative region give rise to rejection decisions, while the boundary rules (coming with the boundary region) result in decisions of non-commitment.

Original rough sets [29] require exact results of classification and do not involve the tolerance for errors. Probabilistic rough sets arise a generalized model for original rough sets, which hinge on two components, i.e. a conditional probability and a pair of thresholds (α, β). The tolerance of errors is characterized by the thresholds of probabilistic rough sets. A series of models of probabilistic rough sets [10,50] was proposed, such as 0.5-probabilistic rough sets [30], decision-theoretic rough sets (DTRS) [48,49], variable precision rough sets (VPRS) [64], Bayesian rough sets [35], parameterized rough sets [9], etc. With respect to probabilistic rough set models, we need to compare the conditional probability with the thresholds in order to make a decision (see Fig. 1).

Following studies reported in [55], the researches on three-way decisions with probabilistic rough sets can be summarized as shown in Fig. 1.

- The determination of threshold values used in probabilistic rough set models. The determination of a pair of thresholds for the probabilistic rough sets constitutes a challenge [22]. The pair of thresholds presented in most probabilistic rough set models

* Corresponding author.
E-mail addresses: decuiliang@126.com (D. Liang), wpedrycz@ualberta.ca (W. Pedrycz), newton83@163.com (D. Liu), huhupei@126.com (P. Hu).

http://dx.doi.org/10.1016/j.asoc.2015.01.008
1568-4946/

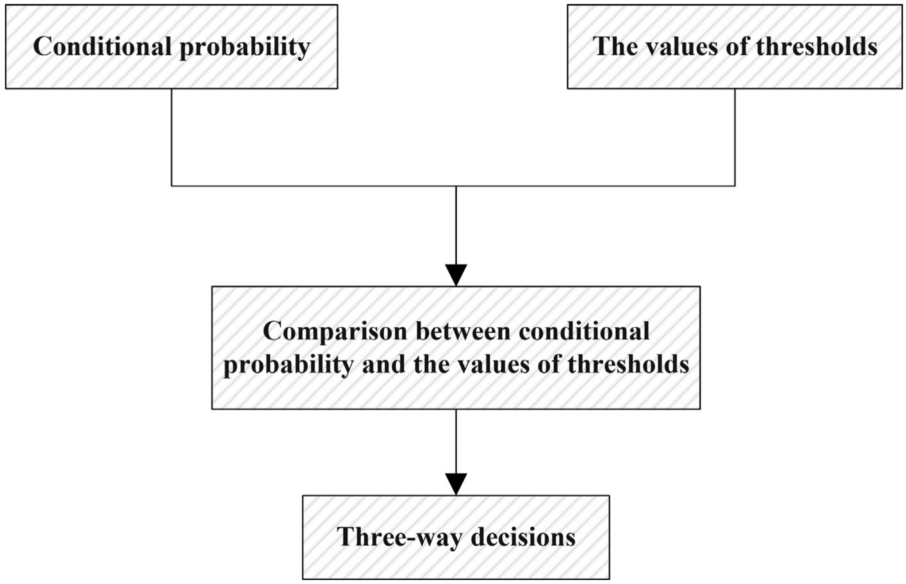

Fig. 1. The main components of three-way decisions augmented by probabilistic rough sets.

comes with subjectively assigned values and lacks of semantics [51,53,54]. In recent years, the determinations of thresholds are discussed in the framework of DTRS, which was first proposed by Yao et al. [48,49] using the Bayesian decision procedure. The pair of thresholds used in the DTRS model can be automatically calculated by loss functions with the minimum expected overall risk. In some cases, the loss function can be expressed in the form of money, energy or time [22]. Yao [51] adopted the relative value of loss function to express the thresholds. Herbert and Yao [11] introduced game theory to determinate the threshold values of probabilistic rough sets. Li and Zhou [17] discussed the value of loss function for a multi-view DTRS decision model based on different attitudes of decision makers. Liang et al. [18] generalized a concept of the precise value of loss function to triangular fuzzy decision-theoretic rough sets. Considering different criteria adopted by different agents, Yang and Yao [47] discussed some aggregation methods of loss functions and proposed a multi-agent DTRS model.

- The determination of the conditional probability used in probabilistic rough set models. Yao and Zhou [52] proposed a naive Bayesian decision-theoretic rough set model, where the conditional probability is estimated by using the Bayes theorem with naive probabilistic independence assumption. Liu et al. [23] employed binary logistic regression to compute the conditional probability of DTRS model.

Evidently, the conditional probability and the thresholds are two important components of three-way decisions. DTRS is a representative model of probabilistic rough sets [48–50]. It not only considers the decision semantics, e.g. cost, risk, but also explains the thresholds of probabilistic rough set models [22,50]. The values of the thresholds used in the probabilistic rough set models are implied by the loss functions and associated with decision makers. Note that the loss functions and the conditional probability presented in the existing studies are mainly numerics. However, in the realistic decision process, some influencing factors result in decision makers not to assign specific numerics, e.g. cost, time and complexity [18]. Under these circumstances, the assessment of the description in terms of quantitative expressions is not suitable, see [39,46]. Other than the results reported in [18], we can resort ourselves to the assessment realized with the aid of linguistic terms. Linguistic terms have been witnessed in numerous situations [2,3,12,25,28,33,37–39,41,42,44,45,58–60]. For example, when evaluating a performance of a smartphone, linguistic terms like *good*, *fair*, *poor* can be considered. When we grade a student, linguistic terms like *good*, *medium*, *poor* can be used. In this case, the linguistic information can be directly used to compute and the linguistic terms need not been transformed into numerical counterparts [7,31,39,41,42,44,45]. Considering the conditional probability and the loss functions of the DTRS model with linguistic terms, we construct a certain type of novel three-way decisions. In light of the results presented in [18], these parameters of three-way decisions are evaluated by decision makers. In order to realize linguistic assessment of both the conditional probability and the loss functions, group decision making [1,4,26,33,37,38,43,61,62] is adopted. Group decision making can aggregate the wisdom of the different domain experts and effectively copes with the risk decision problem. More importantly, it provides a semantic interpretation for the relevant parameters of three-way decisions. Reaching consensus in group decision making becomes an essential step. For the consensus problem of group decision making, Pedrycz and Song [33] developed a novel approach. The approach used a particle swarm optimization (PSO) algorithm to optimize the scale over which linguistic terms were expressed. PSO is a population-based optimization method, which uses a swarm of particles to determine an optimal solution in a search space [6]. The conditional probability and the loss functions comprise the set of multi-attributes. Based on the approach reported in [33], this study further discusses the determination for the values of its parameters (i.e. conditional probability and loss functions) with the aid of multiple-attribute group decision making. The main contribution of our study can be identified as follows: (a) We provide a method of the determination of the two types of parameters used in the DTRS; (b) The application of DTRS is extended to the scenarios of qualitative evaluation; (c) An algorithm is designed to improve the inconsistency of multi-attribute group decision making under linguistic assessment (see Algorithm 1).

The remainder of this paper is organized as follows: Section 2 provides some underlying concepts of probabilistic rough sets, Bayesian decision procedure, DTRS, linguistic terms and related operational laws. In Section 3, a certain type of novel three-way decisions is constructed. With the aid of multi-attribute group decision making with the linguistic assessment, Section 4 designs an adaptive algorithm for consistency improvement and investigates the determination of the values of both the conditional probability and the loss functions. An example is presented to illustrate the