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The complexity of dominating set reconfiguration

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ABSTRACT

Suppose that we are given two dominating sets D_s and D_t of a graph G whose cardinalities are at most a given threshold k. Then, we are asked whether there exists a sequence of dominating sets of G between D_s and D_t such that each dominating set in the sequence is of cardinality at most k and can be obtained from the previous one by either adding or deleting exactly one vertex. This decision problem is known to be PSPACE-complete in general. In this paper, we study the complexity of this problem from the viewpoint of graph classes. We first prove that the problem remains PSPACE-complete even for planar graphs, bounded bandwidth graphs, split graphs, and bipartite graphs. We then give a general scheme to construct linear-time algorithms and show that the problem can be solved in linear time for cographs, forests, and interval graphs. Furthermore, for these tractable cases, we can obtain a desired sequence if it exists such that the number of additions and deletions is bounded by $O(n)$, where n is the number of vertices in the input graph.

1. Introduction

Consider the art gallery problem modeled on graphs: Each vertex corresponds to a room which has a monitoring camera and each edge represents the adjacency of two rooms. Assume that each camera in a room can monitor the room itself and all rooms adjacent to it. Then, we wish to find a subset of cameras that can monitor all rooms; the corresponding vertex subset D of the graph G is called a *dominating set* of G, that is, every vertex in G is either in D or adjacent to a vertex in D. For example, Fig. 1 shows six different dominating sets of the same graph. Given a graph G and a positive integer k, the problem of determining whether G has a dominating set of cardinality at most k is a classical NP-complete problem [9].

1.1. Our problem

However, the art gallery problem could be considered in more "dynamic" situations: In order to temporarily remove cameras for maintenance, we sometimes need to change the current dominating set into another one. To minimize disrup-

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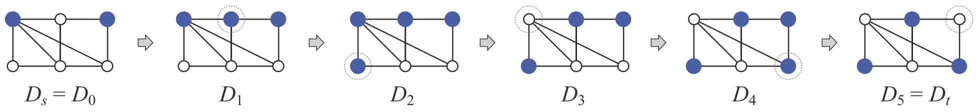

Fig. 1. A sequence $\langle D_0, D_1, \dots, D_5 \rangle$ of dominating sets of the same graph, where $k = 4$ and the vertices in dominating sets are depicted by large (blue) circles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

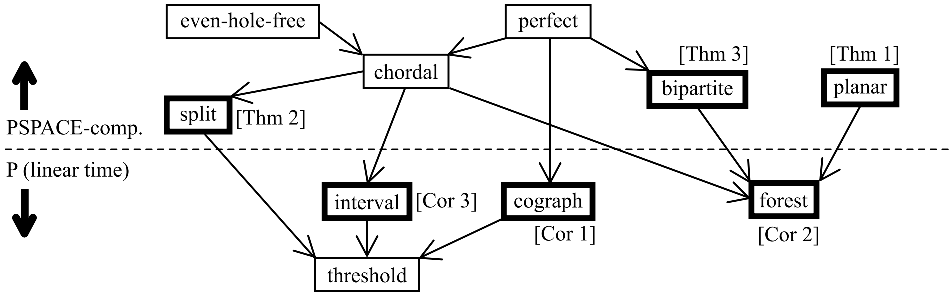

Fig. 2. Our results, where each arrow represents the inclusion relationship between graph classes: $A \to B$ indicates that the graph class B is properly included in the graph class A [6]. We also show PSPACE-completeness on graphs of bounded bandwidth (Theorem 1).

tion, this transformation needs to be done by switching cameras one by one while monitoring all rooms throughout the process.

In this paper, we thus study the following problem: Suppose that we are given two dominating sets of a graph G whose cardinalities are at most a given threshold $k > 0$ (e.g., the leftmost and rightmost ones in Fig. 1, where $k = 4$), and we are asked whether we can transform one into the other via dominating sets of G such that each intermediate dominating set is of cardinality at most k and can be obtained from the previous one by either adding or deleting a single vertex. We call this decision problem the DOMINATING SET RECONFIGURATION problem. For the particular instance of Fig. 1, the answer is yes as illustrated in Fig. 1.

1.2. Known and related results

Similar problems have been extensively studied under the reconfiguration framework [16], which arises when we wish to find a step-by-step transformation between two feasible solutions of a combinatorial problem such that all intermediate solutions are also feasible. The reconfiguration framework has been applied to several well-studied problems, including SATISFIABILITY [10,20,21], INDEPENDENT SET [3,5,8,14,16,19,23,25], VERTEX COVER [16,17,22,23], CLIQUE [16,18], VERTEX-COLORING [2,4,7,13], and so on. (See also a survey [15] and references in [8].)

Mouawad et al. [23] proved that DOMINATING SET RECONFIGURATION is $W[2]$-hard when parameterized by $k + \ell$, where k is the cardinality threshold of dominating sets and ℓ is the length of a desired sequence of dominating sets.

Haas and Seyffarth [11] gave sufficient conditions for the cardinality threshold k for which any two dominating sets can be transformed into one another. They proved that the answer to DOMINATING SET RECONFIGURATION is yes for a graph G with n vertices if $k = n - 1$ and G has a matching of cardinality at least two; they also gave a better sufficient condition when restricted to bipartite and chordal graphs. Suzuki et al. [24] improved the former condition and showed that the answer is yes if $k = n - \mu$ and G has a matching of cardinality at least $\mu + 1$, for any nonnegative integer μ.

1.3. Our contribution

To the best of our knowledge, no algorithmic results are known for DOMINATING SET RECONFIGURATION, and it is therefore desirable to obtain a better understanding of what separates “hard” from “easy” instances. To that end, we study the problem from the viewpoint of graph classes and paint an interesting picture of the boundary between intractability and polynomial-time solvability.[1] (See Fig. 2.)

We first prove that the problem is PSPACE-complete even on planar graphs, bounded bandwidth graphs, split graphs, and bipartite graphs. Our reductions for PSPACE-hardness follow from the classical reductions for proving the NP-hardness of DOMINATING SET [1,9]. However, the reductions should be constructed carefully so that they preserve not only the existence of dominating sets but also the reconfigurability.

We then give a general scheme to construct linear-time algorithms for the problem. As examples of its application, we demonstrate that the problem can be solved in linear time on cographs (also known as P_4-free graphs), forests, and interval

[1] A preliminary version of this paper has been presented in [12].