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Static output-feedback controller design for a fish population system

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A B S T R A C T

This paper examines the problem of output feedback control of a Takagi–Sugeno (TS) fuzzy fishery system. The considered system is the continuous age structured model of an exploited population that includes a nonlinear stock–recruitment relationship. The effort is used as control term, the age classes as states and the quantity of captured fish per unit of effort as measured output. In order to stabilize the stock states around the references equilibrium, which means biologically the sustainability of the fish stock, the output feedback controller is adopted, rather than a controller based on the state observer. An algorithm based on the linear matrix inequality is proposed to compute the static output feedback gain. Simulation results of the continuous fishery systems confirm the effectiveness of the proposed design.

1. Introduction

One of the desirable objectives in the management of fisheries resources is the conservation of the fish population. The formulation of good harvesting policies which take into account this objective is complex and difficult to achieve. It can be realized by stabilizing the stock states around the reference equilibrium, which means biologically the sustainability of the fish stock. In order to solve this control engineering problem several researchers use state feedback as controller. However in fisheries systems, the resources cannot be counted directly, except with acoustic method which is not generalized yet, so the state feedback control law is not realizable. In order to solve this problem, the current paper deals with the synthesis of the output feedback control law to stabilize the states variables around the reference equilibrium [1]. But in [1] the studied model is a structured model with two age classes only, and the application of the Jurdjevic–Quinn [2] method to a model with n $(n > 2)$ age classes is complex. To overcome this limitation, a different technique based on the Takagi–Sugeno (T–S) [3] multimodel approach is introduced, in order to compute the static output feedback gain for this fishery system.

During the last two decades, T–S fuzzy models have attracted the attention of many researchers [4–10]. They offer the possibility to apply some tools coming from the linear theory, whereas those models are composed of linear submodels blended with fuzzy membership functions. The study of stabilization is very often done using a direct Lyapunov approach, and especially with the well-known quadratic functions [11]. T–S fuzzy systems can represent exactly a nonlinear model [11], from this exact model, static output feedback control law may be designed based on the linear subsystems [12].

To the best of our knowledge, the problem of stabilizing exploited fish population systems through static output feedback control law and using T–S fuzzy models has not been studied in the literature. In this work a static output feedback control law based on Takagi–Sugeno multimodel approach is proposed and applied to a continuous nonlinear fish population system. The controller gain is calculated using linear matrix inequalities (LMI) [13].

The outline of this paper is as follows. First, in Section 2, the T–S type fuzzy model is briefly presented to model the fishery system, and based on Lyapunov's approach; a stability criterion is derived to guarantee the stability of the fishery system via an LMI formulation. Then, Section 3 deals with the description of the continuous stage structured model, which is transformed to a T–S fuzzy model. In Section 4, the procedure to design the control law is applied, and simulations using numerical data of some fisheries models are given to demonstrate the effectiveness of the proposed controller.

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2. Takagi–Sugeno fuzzy models

2.1. Model representation

In the following the concept of T–S multiple model is introduced. The main idea of the T–S fuzzy modeling method is to partition the nonlinear system dynamics into several locally linearized subsystems, so that the overall nonlinear behavior of the system can be captured by fuzzy blending of such subsystems through nonlinear fuzzy membership functions. Unlike conventional modeling techniques which use a single model to describe the global behavior of a nonlinear system, fuzzy modeling is essentially a multi-model approach in which simple submodels (typically linear models) are fuzzily combined to describe the global behavior of a nonlinear system.

The continuous-time T–S fuzzy system is described as a set of N rules, where each rule i uses p membership functions $(M_{i1}, \ldots, M_{ip})$ and p fuzzy variables $(z_1(t), \ldots, z_p(t))$, as follows:

Model Rule i :

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$

$\dot{y}(t) = C_i x(t)$

Here, M_{ij} is the fuzzy set, r is the number of model rules, $x(t) \in \boldsymbol{R}^n$ is the state vector, $u(t) \in \boldsymbol{R}^m$ is the input vector, $y(t) \in \boldsymbol{R}^q$ is the output vector, $A_i \in \boldsymbol{R}^{n\times n}$, $B \in \boldsymbol{R}^{n\times m}$, and $C \in \boldsymbol{R}^{q\times n}$; $z_1(t), \ldots, z_p(t)$ are known premise variables that may be functions of the state variables, external disturbances, and/or time.$z(t)$ will be used to denote the vector containing all the individual elements $z_1(t), \ldots, z_p(t)$. Given a pair of $(x(t), u(t))$, and using singleton fuzzifier, max-product inference and center average defuzzifier, the aggregated fuzzy model can be written as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i\{z(t)\}\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} w_i(z(t))} \quad (1)$$

where $z(t) = [z_1(t) z_2(t) \ldots z_p(t)]$, and $w_i\{z(t)\} = \prod_{j=1}^{p} M_{ij}\{z_j(t)\}$.

The term M_{ij} $\{z_j(t)\}$ is called the membership function. It is the grade of membership of $z_j(t)$ in M_{ij}. Eq. (1) can be written as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i\{z(t)\}\{A_i x(t) + B_i u(t)\} \quad (2)$$

where $\mu_i\{z(t)\} = w_i\{z(t)\}/\sum_{j=1}^{r} w_j\{z(t)\}$. μ_i $\{z(t)\}$ is called the activation function.

Since $\sum_{i=1}^{r} w_i\{z(t)\} > 0$ and $w_i\{z(t)\} \geq 0$, $i = 1, 2, \ldots r$, one has: $\sum_{i=1}^{r} \mu i\{z(t)\} = 1$ and μ_i $\{z(t)\} \geq 0$, $i = 1, 2, \ldots, r$, for all t.

The global output of T–S model is interpolated as follows:

$$y(t) = \sum_{i=1}^{r} \mu_i\{z(t)\} C_i x(t) \quad (3)$$

2.2. Stability conditions and control design

An LMI-based design method using fuzzy state feedback control has been proposed in [14]. However, in real-world control problems, the states may not be completely accessible. In such cases, one needs to resort to output feedback design methods that are useful when only the output of the system is available. An attempt to solve this problem was the synthesis of the static output feedback control law to stabilize the state variables around the reference equilibrium [1]. But in [1] the studied model is a structured model of two age classes only, and the application of the Jurdjevic–Quinn [2] method to a model of n ($n > 2$) age classes is complex. Thus, fuzzy static output feedback control is the most desirable since it can be implemented easily with low cost.

In the literature, the main control law used as a nonlinear static output feedback is the output parallel distributed compensation (OPDC). In the OPDC synthesis, each control rule is designed from the corresponding rule of a T–S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy models (3) the following fuzzy controller via the OPDC law is constructed:

Control Rule i :

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}

THEN $u(t) = F_i y(t) \quad i = 1, 2, \ldots, r$

The overall fuzzy control law is composed of several linear output feedbacks blended together using the nonlinear functions $\mu_i(\cdot)$ of the model:

$$u(t) = \sum_{i=1}^{r} \mu_i\{z(t)\} F_i y(t) \quad (4)$$

The fuzzy controller design is to find the local feedback gains $F_i \in \boldsymbol{R}^{mq}$ in the consequent parts.

In the sequel, it is assumed that $C_i = C$, $i = 1, \ldots, r$, is full row rank. By substituting (4) into (2), the closed-loop fuzzy system under consideration is:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i\{z(t)\}\mu_i\{z(t)\}\{(A_i + B_i F_i C)x(t)\} \quad (5)$$

The stability of the closed-loop system (5) has been investigated in the literature: see [11] and the references there in. Here, a previous result on the stabilization of continuous-time fuzzy systems that are obtained via a quadratic Lyapunov function [12] is announced. It deals with sufficient conditions in LMIs form to ensure asymptotic stability of (5).

Theorem 1 ([12]). Suppose that there exist matrices N_i, M, S and Q such that

$$\begin{aligned} & Q > 0, \quad S > 0 \\ & QA_i^T + A_i Q + C^T N_i^T + B_i N_i C + (s-1)S < 0 \quad i = 1, 2, \ldots, r \end{aligned} \quad (6a)$$

$$\begin{aligned} Q(A_i + A_j)^T &+ (A_i + A_j)Q + C^T(N_j^T B_j^T) \\ &+ (B_i N_i + B_j N_i)C - 2s \leq 0 \quad i = 1, 2, \ldots, r \end{aligned} \quad (6b)$$

and

$$CQ = MC \quad (7)$$

with $\mu_i(z(t))\mu_i(z(t)) \neq 0$. Then the T–S model (5) is globally asymptotically stable with the OPDC controller (4) where

$$F_i = N_i M^{-1} \quad \forall i \in \{1, \ldots, r\} \quad (8)$$

s is the number of submodels simultaneously activated.

Remarks.

- Note that the design conditions presented above (Theorem 1) are only sufficient conditions. A major advantage of these conditions compared especially with the application of the Jurdjevic–Quinn [2] method used in [1] is that they are cast into an LMI form, and therefore easily solvable.
- Theorem 1 overcomes the source of conservatism for the proposed design in [1]. Obtained results are not valid in an invariant domain only.