Theoretical Computer Science 647 (2016) 50–58

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

# A new design formula exploited for accelerating Zhang neural network and its application to time-varying matrix inversion ☆

Lin Xiao

*College of Information Science and Engineering, Jishou University, Jishou 416000, China*

ARTICLE INFO

*Article history:*
Received 8 January 2016
Received in revised form 21 June 2016
Accepted 21 July 2016
Available online 27 July 2016
Communicated by R. Gavaldà

*Keywords:*
Design formula
Time-varying matrix inversion
Zhang neural network
Theoretical analysis
Simulation verification

ABSTRACT

Online solution to time-varying matrix inverse is further investigated by proposing a new design formula, which can accelerate Zhang neural network (ZNN) to finite-time convergence. Compared with the existing recurrent neural networks [e.g., the gradient neural networks (GNN), and the original Zhang neural network], the proposed neural network (termed finite-time ZNN, FTZNN) makes a breakthrough in the convergence performance (i.e., from infinite time to finite time). In addition, different from the previous processing method (i.e., choosing a better nonlinear activation to accelerate convergence speed), this paper subtly proposes a new design formula to accelerate the original ZNN model and design the new FTZNN model. Besides, theoretical analyses of the design formula and the FTZNN model are given in detail. Simulative results substantiate the effectiveness and superiorness of the proposed FTZNN model for online time-varying matrix inversion, as compared with the GNN model and the original ZNN model.



## 1. Introduction

The online inversion of a matrix arises in many scientific fields and finds wide engineering applications; e.g., in MIMO systems [1,2], graph estimation [3], cloud computing [4], and robot kinematics [5,6]. Besides, as an essential step of many solutions, when we want to understand the physical mechanism of phenomena, exact solutions for the matrix inversion have to be obtained [7]. Thus, much effort has been devoted to the fast and high accuracy solution of such a matrix inversion problem, and various methods have been presented and investigated for matrix inversion [4,5,8–11].

Generally speaking, the methods of the computing matrix inverses can be classified into two categories: those based on discrete models and those based on continuous models. To begin with, many discrete algorithms have been presented to approximately find the inverse of matrix [4,8–11]. Specifically, these algorithms start with a given initial value and recursively update the estimate to improve the approximate solution until an approximation to the theoretical solution is obtained with a desired accuracy. However, computing matrix inverse is a demanding task, and the difficulty is significantly amplified when the inverse has to be obtained in real time. In view of the fact that the minimal arithmetic operations of discrete algorithms are usually proportional to the cube of the matrix dimension [12], various continuous models have been presented and investigated to remedy the weaknesses of discrete models for matrix inversion [13–19].

☆ This work is supported by the National Natural Science Foundation of China (grant no. 61503152), the Natural Science Foundation of Hunan Province, China (grant no. 2016JJ2101), the Research Foundation of Education Bureau of Hunan Province, China (grant no. 15B192), and the National Natural Science Foundation of China (grant nos. 61563017, 61561022, 61363073 and 61363033).

*E-mail address:* xiaolin860728@163.com.

http://dx.doi.org/10.1016/j.tcs.2016.07.024
0304-3975/

As typical continuous models, recurrent neural networks (RNNs) are becoming promising approaches to solve matrix inversion and optimization problems [20–22]. One appealing feature of RNNs lies in the parallel processing and hardware implement ability, as compared to discrete models. Especially, past three decades witnessed the birth and prosperity of RNNs. Recent developments in VLSI, FPGA, DSP and optical chips make the implementation of RNNs more feasible at a more reasonable cost [23]. As a kind of typical RNNs, gradient-based neural networks (GNNs) are particularly developed for solving static matrix inversion problem [13,14]. For example, in [14], a GNN model was presented and investigated for online matrix inversion with detailed theoretical analysis given. This kind of approach uses the norm of the error matrix as the performance indicator and designs a neural network evolving along the negative gradient-descent direction to make the error norm decrease to zero with time. However, for the time-varying case, the error norm cannot converge to zero even after infinitely long time due to the lack of velocity compensation of time varying parameters [15–19]. For avoiding the lagging errors generated by the GNN model, a special kind of RNN (termed the original Zhang neural network, ZNN) was proposed and investigated for time-varying matrix inversion [15–19]. It is theoretically proved that the original ZNN model can converge to the theoretical time-varying solution with time. Besides, from [19], we can conclude that, when used to solve the static or time-varying matrix inversion problem, the convergence performance of the original ZNN model is a significant improvement, as compared to the GNN model.

Due to in-depth research on the original ZNN model, we found that the finite-time convergence to the theoretical time-varying solution cannot be realized by the original ZNN model, so that its applications in real-time calculation may be limited [24–29]. Therefore, how to improve the convergence speed is always of importance for the original ZNN model. Inspired by the study on the finite-time convergence of continuous autonomous system [24,25], we find that the term $E^{q/p}(t)$ can make the ordinary differential equation converge to the equilibrium state within finite time. Keeping this point in mind, we try to improve the convergence speed of the original ZNN model by adding the term $E^{q/p}(t)$ to the original design formula $\dot{E}(t) = -\gamma\Phi(E(t))$. It is worth mentioning out that the convergence rate of neural-network models can be accelerated by choosing a proper nonlinear activation function, because a properly-designed nonlinear activation function often outperforms the linear one in convergence rate [26–29]. In this paper, different from the previous processing method (i.e., choosing a better nonlinear activation to accelerate convergence speed), a new design formula is subtly proposed to accelerate the original ZNN model and used to design a finite-time ZNN (FTZNN) model for time-varying matrix inversion. In addition, theoretical analyses of the design formula and the FTZNN model are carried out to show the effectiveness and superiorness of the proposed method. To the best of the author's knowledge, this is the first time to present such a new design formula and the FTZNN model for solving time-varying matrix inversion problem. Before ending this section, it is worth listing the main contributions of this paper as follows.

1) For the first time, a novel design formula is proposed and used to construct the FTZNN model for solving online time-varying matrix inversion problem.
2) It can be guaranteed theoretically that the proposed FTZNN model globally converges to the theoretical time-varying solution within finite time. In addition, the upper bound of the finite convergence time for the FTZNN model is derived analytically in theory.
3) For comparison purposes, the GNN model and the original ZNN model are also applied to time-varying matrix inversion. Comparative results show that the FTZNN model outperforms the original ZNN model and the GNN model, with finite-time convergence achieved.

## 2. Problem formulation and preliminaries

The online matrix inversion is widely encountered in various engineering and scientific fields [1–5]. In mathematics, the time-varying case of matrix inversion can be defined as below:

$$A(t)X(t) = I \in \mathbb{R}^{n\times n}, \text{ or } X(t)A(t) = I \in \mathbb{R}^{n\times n}, \tag{1}$$

where $t$ stands for time, $X(t) \in \mathbb{R}^{n\times n}$ is an unknown time-varying matrix to be obtained, $A(t) \in \mathbb{R}^{n\times n}$ denotes a known time-varying coefficient matrix, and $I \in \mathbb{R}^{n\times n}$ denotes the identity matrix of an appropriate size. In this paper, we limit the discussion to the situation where $A(t)$ is nonsingular at any time instant $t \in [0, +\infty)$ so that (1) has an unique time-varying solution. Without loss of generality, let $X^*(t) \in \mathbb{R}^{n\times n}$ denote the time-varying theoretical solution of (1).

For comparative purposes, the gradient-based neural network (GNN) for time-varying matrix inversion is directly presented as follows [13,14]:

$$\dot{X}(t) = -\gamma A^{\mathrm{T}}(t)\left(A(t)X(t) - I\right), \tag{2}$$

where $A^{\mathrm{T}}(t)$ denotes the transpose of time-varying coefficient matrix $A(t)$, and design parameter $\gamma > 0$ is used to adjust the convergence rate of the GNN model (2). Besides, by following Zhang et al.'s design method, the original Zhang neural network (ZNN) for time-varying matrix inversion can be given out directly as follows [15,16]:

$$A(t)\dot{X}(t) = -\gamma\Phi(A(t)X(t) - I) - \dot{A}(t)X(t), \tag{3}$$

where $\Phi(\cdot)$ denotes an activation function array. For the original ZNN model (3), we have the following lemma to guarantee its exponential convergence when applied to time-varying matrix inversion [15,16].