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Celebrity games

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ABSTRACT

We introduce *Celebrity games*, a new model of network creation games. In this model players have weights (W being the sum of all the player's weights) and there is a critical distance β as well as a link cost α. The cost incurred by a player depends on the cost of establishing links to other players and on the sum of the weights of those players that remain farther than the critical distance. Intuitively, the aim of any player is to be relatively close (at a distance less than β) from the rest of players, mainly of those having high weights. The main features of celebrity games are that: computing the best response of a player is NP-hard if $\beta > 1$ and polynomial time solvable otherwise; they always have a pure Nash equilibrium; the family of celebrity games having a connected Nash equilibrium is characterized (the so called *star celebrity games*) and bounds on the diameter of the resulting equilibrium graphs are given; a special case of star celebrity games shares its set of Nash equilibrium profiles with the MaxBD games with uniform bounded distance β introduced in Bilò et al. [6]. Moreover, we analyze the Price of Anarchy (PoA) and of Stability (PoS) of celebrity games and give several bounds. These are that: for non-star celebrity games PoA = PoS = $max\{1, W/\alpha\}$; for star celebrity games PoS = 1 and PoA = $O(min\{n/\beta, W\alpha\})$ but if the Nash Equilibrium is a tree then the PoA is $O(1)$; finally, when $\beta = 1$ the PoA is at most 2. The upper bounds on the PoA are complemented with some lower bounds for $\beta = 2$.

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1. Introduction

The global growth of Internet and social networks usage has been accompanied by an increasing interest to model theoretically their creation as well as their behavior. In particular, network creation games (NCG) aim to model Social Networks and Internet by simulating the creation of a decentralized and non-cooperative communication network among n players (the network nodes).

From the seminal paper [14] several proposals have been made in the area of NCG. In the original model, the goal of each player is to have, in the resulting network, all the other nodes as close as possible while buying as few links as possible [14]. Several assumptions are made: all the players have the same interest (all-to-all communication pattern with identical weights); the cost of being disconnected is infinite; and the edges paid by one node can be used by others. Formally, a game Γ in this model is defined as a tuple $\Gamma = \langle V, \alpha \rangle$, where V is the set of n nodes and α the cost of establishing a link. A strategy for player $u \in V$ is a subset $S_u \subseteq V - \{u\}$, the set of players for which player u pays for

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establishing a link. The n players and their joint strategy choices $S = (S_u)_{u \in V}$ create an undirected graph $G[S]$. The cost function for each node u under strategy S is defined by $c_u(S) = \alpha|S_u| + \sum_{v \in V} d_{G[s]}(u, v)$ where $d_{G[s]}(u, v)$ is the distance between nodes u and v in graph $G[S]$. Because of the summation in the cost function this model is informally known as the *Sum game* model. By changing the cost function to $c_u(S) = \alpha|s_u| + \max\{d_{G[S]}(u, v) | v \in V\}$ as proposed in [12] one obtains the *Max game* model.

From here on several versions and variants have been considered. Instead of buying links unilaterally, Corbo and Parkes [9] proposed the possibility of having links formed by bilateral contracting: both endpoints must agree before creating a link between them and the two players share (half–half) the cost of establishing the link. NCG models can be cooperative – a possibility introduced by Albers et al. [1] – and therefore any node can purchase any amount of any link in the resulting graph, and a link can be created when its cost is covered by a set of players. The model studied in [7] (see also [6]) considers the notion of bounded distance per player and proposes two variants: the MaxBD game and the SumBD game, corresponding to the original Max and Sum cost models respectively. The cost in those games depends on whether the player's eccentricity is smaller or equal than the associated bounded distance. In that case a player pays the number of established links, otherwise its cost is infinite. For further variants we refer the interested reader to [2,3,5,8,10–13,15,16,18] among others.

We introduce *celebrity games* a NCG where players have different weights and share a common distance bound. As far as we understand, not all the nodes in Internet based networks have the same importance. It is though natural to consider players with different relevance weights. In such a setting, the cost of being far (even if connected) from important nodes (the ones with high weight) should be higher than the cost of having them close. Intuitively, the goal of each player in celebrity games is to buy as few links as possible in order to have the high-weighted nodes (or groups of nodes) closer to the given critical distance. Observe that if the cost of establishing links is higher than the benefit of having close a node (or set of nodes), players might rather prefer to stay either far or even disconnected from it.

Our aim is to study the combined effect of having players with different weights that share a common bounded distance. Although heterogeneous players have been considered recently in the context of NCG under bilateral contracting [4,17], and Bilò et al. [7] consider the notion of bounded distance, to the best of our knowledge this is the first model that studies how a common critical distance, different weights, and a link cost, altogether affect the individual preferences of the players.

In our model the cost of a player has two components. The first one is the cost of the links established by the node. The second one is the sum of the weights of those nodes that are farther away than the critical distance. More specifically, the parameters of a celebrity game are: a weight to each player; a cost for establishing a link; and a critical distance. Formally, a celebrity game is defined by $\Gamma = \langle V, (w_u)_{u \in V}, \alpha, \beta \rangle$, where V is a set of nodes with weights $(w_u)_{u \in V}$, α is the cost of establishing a link and, β establishes the desirable distance bound. Celebrity games include the MaxBD games introduced in [7] (see Section 5 for the details). They capture not only the cases in which players are indistinguishable but those cases where the players may have different weights affecting differently the costs of the other players.

We analyze the structural properties of the Nash equilibrium (NE) graphs of celebrity games and their quality with respect to the optimal strategies under the usual social cost. To do so we address the cases $\beta = 1$ and $\beta > 1$ separately. Notice that, for $\beta = 1$, each player u has to decide for every non-edge (u, v) of the graph to pay either α for the link or w_v (the weight of the non-adjacent node v) while, for $\beta > 1$, every player u has to choose for each non-edge (u, v) between buying the link (u, v) and paying α minus the sum of the weights of those nodes whose distance to u will become less or equal than the critical distance β or paying the sum of the weights of the nodes with distance to u greater than β.

For the general case $\beta > 1$ our results can be summarized as follows:

- Computing a best response for a player is NP-hard.
- The optimal social cost of a celebrity game Γ depends on the relation between the total sum of the weights W and the cost α of buying a link: $\text{OPT}(\Gamma) = \min\{\alpha, W\}(n-1)$. Nevertheless, pure NE always exist and NE graphs are either connected or a set of isolated nodes. Again, the relationship between the cost of establishing a link and the weight of the nodes leads to different types of NE.
- We use the term *celebrity* for a node whose weight is strictly greater than the cost of establishing a link. Having at least one celebrity guarantees that all NE graphs are connected, although there are celebrity games without celebrities that still have connected NE graphs. In those games having a connected NE graph, a star tree is always a NE graph. We called this subfamily of celebrity games *star celebrity games*.
- For star celebrity games, we obtain a general upper bound of $2\beta + 1$ for the diameter of NE graphs. In particular, if G is a NE tree we show that $diam(G) \leq \beta + 1$, otherwise $\beta/2 < diam(G) \leq 2\beta + 1$. The upper bound can be improved by considering the relationship between α and the maximum and minimum weights, w_{max} and w_{min}, respectively. So, if $w_{min} \leq \alpha < w_{max}$, then $diam(G) \leq 2\beta$. On the contrary, if $\alpha < w_{min}$, then $diam(G) \leq \beta$.
- For star celebrity games with $\alpha < w_{min}$, we show that the set of NE strategy profiles coincides with the set of NE strategy profiles of a MaxBD game with uniform bounded distance β.
- We find several bounds on the Price of Anarchy (PoA) and of stability (PoS). For non-star celebrity games PoS = PoA = $\max\{1, W/\alpha\}$. For star celebrity games the PoS is 1 and we obtain a general upper bound of $O(\min\{n/\beta, W/\alpha\})$ for the PoA. We also show particular games on n players having PoA = $\Omega(n)$, for $\beta = 2$. To complement those results we prove that the PoA on NE trees is constant (special cases like trees are also considered in the literature, see for instance [2,3,13].

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