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Tolerance analysis — Form defects modeling and simulation by modal decomposition and optimization[☆]

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ABSTRACT

Tolerance analysis aims on checking whether specified tolerances enable functional and assembly requirements. The tolerance analysis approaches discussed in literature are generally assumed without the consideration of parts' form defects. This paper presents a new model to consider the form defects in an assembly simulation. A Metric Modal Decomposition (MMD) method is henceforth, developed to model the form defects of various parts in a mechanism. The assemblies including form defects are further assessed using mathematical optimization. The optimization involves two models of surfaces: real model and difference surface-base method, and introduces the concept of signed distance. The optimization algorithms are then compared in terms of time consumption and accuracy. To illustrate the methods and their respective applications, a simplified over-constrained industrial mechanism in three dimensions is also used as a case study.

1. Introduction

A manufactured product always has different geometric features from the defined nominal characteristics irrespective of the manufacturing processes employed and the materials used. Tolerance analysis aims to subsequently study the influence of such geometric variations within the obtained features on the behavior of a mechanical system by checking if the geometric tolerances of the components ensure the compliance of a mechanical system in terms of functional requirements. Tolerances analysis also has an impact on all the stages of a product lifecycle such as designing, process planning, etc. and allows to improve the product quality by reducing the associated cost and ensuring reliability [1,2].

Traditionally, there are two major types of tolerance modeling: statistical estimation and worst-case estimation. By applying the statistical analysis (e.g., based on Monte-Carlo simulation), the tolerances can ensure acceptability of a certain large number of assemblies [3,4]. The worst-case methods, on the other hand, generate a functional assembly from the combination of dimensions and tolerances of individual components. Therefore, the tolerances which satisfy the worst case method ensure 100% acceptability of the assemblies [5].

Moreover, tolerancing activities, when simulating geometric deviations, are generally based on the hypothesis that parts are ideals and surfaces have no form defects [4–7]. The consideration of parts' form defects is integral to the tolerancing activities and hence, make it significant to highlight its impacts on the assembly probability estimation in the product quality and cost assessment [8,9]. Grandjean et al. [10] mentioned that the non-consideration of form errors could lead to noncompliant assemblies even if all parts respect the geometrical specifications. Generally, the need to predict and manage all deviations is an important issue in product design, tolerance synthesis and assembly simulation [11]. Globally, there are three main issues in tolerance analysis: geometrical defects modeling, geometrical behavior modeling and technical solutions determining. Each of the issues is explained in the following paragraphs:

Geometrical defects modeling

For geometrical defects modeling, real surfaces derived from a manufacturing process, can be modeled by substituted surfaces [12]. A 'substituted surface' is an ideal surface (geometrically perfect) that is not only the same as the nominal surface but also characterizes a specific physical realization. Furthermore, the substituted-surface model is used to simulate geometric defects in a real surface by situation and dimension deviations. 'Situations deviations' can be mathematically represented with the help of the vectors [13], by a small displacement torsor SDT [12], by a matrix [14] or by a metric torsor [15]. Also, the deviations between two surfaces potentially in contact can be formalized based on the

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SDT and substituted models cannot take form defects into account when characterizing geometrical defects.

Many authors proposed different techniques to geometrically model the defects. For example, Morière [16] proposed to use polynomial functions of degree '2' to model surfaces with defects. Merkley [17] also developed the use of random Bezier curves to model the geometrical defects wherein shape deviations were parameterized by constraining the displacement of the control points of Bezier curves. Lagrange polynomials (or splines) and Bezier surfaces including parts' form defects are traditionally considered in Computer Aided Design [18]. Franciosa et al. [19] presented a morphing mesh-based approach to generate variational shapes according to a small set of points in the nominal geometries of parts. From the mesh model of parts, the associated nodes were moved by applying a morphing procedure. Deviation points, called control points, then defined the concerning local deformations of the surfaces of the parts.

Another concept for modeling parts deviations is the 'Skin Model' which was introduced by Ballu and Mathieu to provide a global representation of the parts' surfaces and acts to express geometric specifications [20,21]. The concept stemmed from the theoretical foundations of Geometrical Product Specification (GPS) and aimed to enrich the nominal idealized geometry considering physical shapes. The Skin model also represents the interface of the part with its environment [22]. It has been used in some recent works by Schleich et al. [11,23], for the assembly simulation of over-constrained systems. Skin Model shapes were used by Schleich et al. [24] as a finite model to illustrate the specific realizations of the skin model in conjunction with geometric deviations being resulted from manufacturing and assembly processes. In short, the skin model shapes are specific outcomes of the skin model employing discrete geometry representation schemes, such as point clouds and surface meshes. Based on this concept, an assembly simulation approach integrating point-based Skin Model Shapes, along with mathematical constrained optimization and difference surface definition, are proposed [23].

In addition to the above discussed issue of modeling geometrical part deviation, some recent works which deal with the decomposition and reconstruction of parts' form defects of simple geometry (e.g., cylinder, plane), have also been developed. Discrete-Cosine-Transformation based decomposition method was proposed by Huang et al. [25] and decomposed the defect into a series of independent modes where each mode can represent various manufacturing defect patterns. For example, the deformation mode can represent part distortion during stamping operations. The method has been adopted by Lecompte et al. [26] to predict the form defects of plane surface as a sum of individual technological defects while the orientation and position defects were not taken into account. Moreover, it was extracted that this method worked well just on rectangular-based surface. Fourier series decomposition method based on Discrete Fourier Transform (DFT) was then introduced by Raja and Radhakrishnan [27] to model form defects which could be applied in most cases to discrete objects. The shape of the geometry can be thus be reconstructed by DFT inverse. Henke et al. [28] also used the Tchebychef Fourier Series model to describe the forms of a cylinder and identify specific types of error shapes. A model of eigen-shapes derived from a Principal Component Analysis (PCA) was also proposed considering the information from measurements to simulate shapes [29].

Samper et al. [30] developed the Discrete Modal Decomposition (DMD) which was initiated on the correlation between sound vibration of a bell and geometric defects. The DMD is based on the natural mode shapes of a discretized feature. The modes are generated by a finite element mesh and a modal basis can then be built. Each defect is characterized as a combination of elementary modes that model the geometric shape. To achieve that, the initial geometric element is discretized and transformed into a discrete mechanical structure. The structure is then modeled by a stiffness matrix **K** and a mass matrix **M**. The modes are obtained by solving the differential equation of a conservative system; $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0$, introducing the two matrices. Modal rebuilding then provides the surface with form defects and a surface **T** can be represented as follows:

$$\mathbf{T} = \sum_{i=1}^{n} \lambda_i Q_i \tag{1}$$

where n defines the number of modes, Q_i are the elementary modes characterizing the surface with form defects, and λ_i are the amplitudes of modes. The method requires a large amount of measurement data and a particular modal solver.

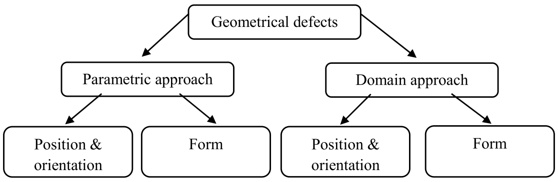

Fig. 1. Geometrical defects modeling.

Fig. 1 depicts a classification scheme of the geometrical defects modeling detailed in this section. The substituted surface dealt with position and orientation defects and is generally considered part of the domain approaches. Domain approaches are defined when situations are handled with mathematical constraints or equations. Skin model, DCT, DFT and DMD take form defects of parts into consideration especially when developing parametric approaches and this was implemented in various commercial tools for tolerance analysis.

Geometrical behavior modeling

Behavior modeling involves the definition of the mathematical models to characterize the system behavior with deviations by knowing how features of systems interact. The mathematical formulations depend on a diagram (called Joint Graph) describing the features, the links between them (with or without gaps), and the functional conditions to specify the global topological structure of the mechanical systems through dimensional chains [31]. Moreover, the mathematical formulations are defined by: (i) equations which constraint part deviations, gaps and functional characteristics; (ii) inequalities which constraint functional requirements, and (iii) inequalities and equations which constraint gaps. Here, the equations define the relations of displacements in the different loops of the joint graph, the relations of displacements represent the linear compatibility constraints between deviations and gaps in different loops, while the inequalities and equations define the interface constraints that characterize the non-interferences between surfaces that are nominally in contact with each other [32–36]. Nevertheless, considering positional deviations in a 3-dimensional context could lead to highly non-linear functions which then have to be linearized piecewise [7,37].

Functional requirements which are represented by inequalities limit the displacements (orientation and location) between surfaces in functional condition. To ensure these requirements, all inequalities of non-interference and compatibility equations must be respected. Recently, Lê et al. [38] proposed the concept of a gap hull and the introduction of difference surface to study the behavior of a planar joint when surfaces are with form defects. Some other studies give matting solutions for circular [39] or prismatic joints [8].