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Isogeometric segmentation: Construction of cutting surfaces☆

Michael Haberleitner *, Bert Jüttler

Altenberger Str. 69, 4040 Linz, Institute of Applied Geometry, Johannes Kepler Universität Linz, Austria

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ABSTRACT

The objective of Isogeometric Segmentation is to generate a decomposition of a solid, given in boundary representation, into a collection of a relatively small number of base solids, which can easily be subdivided into topological hexahedra. This can be achieved by repeatedly splitting the solid. In each splitting step, one chooses a cutting loop, which is a cycle of curves around the boundary of the solid, and constructs a cutting surface that splits the solid into two simpler ones. When only hexahedra or pre-defined base solids are left this process terminates.

The construction of the cutting surface must ensure that two essential properties are fulfilled: the boundary curves of the surface interpolate the previously constructed cutting loop and the surface neither intersects itself nor the boundary of the solid. A novel method for generating the cutting surface is presented in this paper. The method combines two steps: First we generate an implicit guiding surface, which is subsequently approximated by a trimmed spline surface in the second step.

1. Introduction

Since its introduction by T.J.R. Hughes et al. in 2005 [1], the framework of isogeometric analysis has attracted rapidly growing attention from the numerical analysis and the geometric modeling communities. The underlying idea, namely to combine finite element analysis with geometric design by reusing the same basis functions, has led to significant improvements of the interaction between the representations used in Computer-Aided Design and in Numerical Simulation, see [2] for more information. The current state-of-the-art in this field is captured by the two recent special issues of influential journals [3,4].

With the growing interest in isogeometric analysis, it was soon noticed that the realization of its potential advantages requires to address new challenging problems. A prominent example is the need to develop techniques for creating NURBS-based domain parameterizations from boundary-represented CAD data, as the resulting NURBS representations provide the basic description of geometric data for isogeometric analysis, cf. [5].

These parameterizations may be classified into single- and multi-patch representations. The construction of single patch spline models from boundary data has been addressed by numerous publications. We briefly mention some of them: Gravesen et al. address the challenge of creating a regular single-patch domain parameterization from boundary data [6]. A method for volumetric parameterization and trivariate B-spline fitting using harmonic mappings has been described by Martin et al. [7] for objects of cylindrical topology. Zhang et al. [8] describe a construction of a solid T-spline parameterization for genus zero objects from triangulated boundary data. This method has later been extended to solids possessing an arbitrary topological genus [9].

Although it has some benefits, the use of a single patch imposes severe constraints on the topology of the domain. Algorithms for creating multi-patch representations, that provide increased flexibility, are therefore of vital interest. Typically, such algorithms consist of two steps: First the domain is subdivided into a collection of topological hexahedra. Second, one constructs a spline parameterization for each of these blocks. In order to benefit from the potential advantages of isogeometric analysis, one should construct segmentations into relatively few hexahedral patches. This is quite different from the usual approach to hexahedral mesh generation, which has been studied in the context of the classical finite element method, see e.g. [10] and the references cited therein.

Parameterization techniques for multi-patch domains have been studied by Xu et al. [11] using variational methods. A combinatorial approach to planar multi-patch domains, which is based on a complete enumeration of the possible patch layouts, has been described recently in [12]. Suitable spline spaces for multi-patch domains have been analyzed in [13,14].

The problem of decomposing a domain into a small number of topological hexahedra, which are suitable for spline parameterizations, has been called the isogeometric segmentation problem in [15]. One may distinguish between two approaches:

The first one uses splines on polycube domains, see e.g. [16,17]. The parameterization algorithm first generates a polycube domain

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* Corresponding author.
E-mail addresses: michael.haberleitner@jku.at (M. Haberleitner), bert.juettler@jku.at (B. Jüttler).

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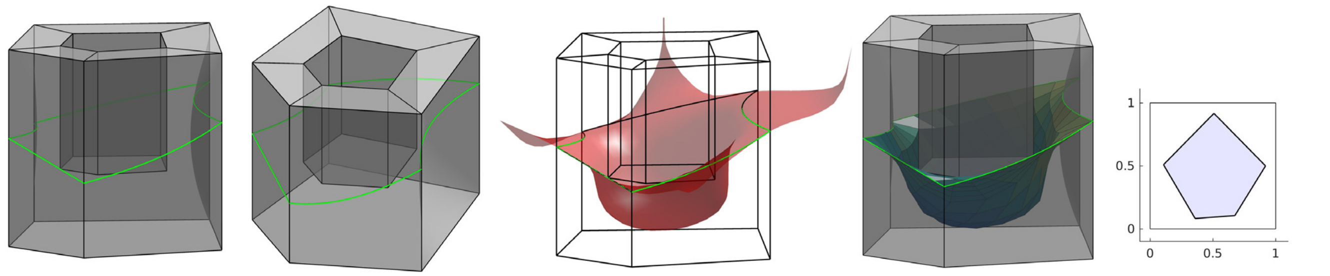

Fig. 1. From left to right: Vase-shaped object with non-planar faces seen from two different viewing directions, implicit guiding surface, parameterized surface patch, and its automatically created parameter domain.

(i.e., a collection of cubes) that resembles the given solid object, and constructs a parameterization by considering a deformation that transforms the domain into the solid. Al Akhras et al. combine polycubes with pants decomposition of the boundary surface to subdivide the given solid into a collection of cuboids [18]. Clearly, the polycube-based approach is quite powerful but has some difficulties when dealing with features (sharp edges) on the boundary. This problem has been addressed recently in [19].

The second approach, which has been established in a series of papers [15,20,21], is based on iterated splitting of the initial solid using cutting surfaces. This surface is obtained from a cutting loop, which is a cycle of curves on the solid's boundary surface. While the selection of the loop and the construction of its curve segments is now well understood, the actual construction of the cutting surface has not yet been investigated.

The current paper focuses on this problem, which is an essential ingredient of the isogeometric segmentation pipeline described in [22]. Given a three-dimensional solid in boundary representation as a collection of trimmed NURBS surfaces, and a cutting loop, we generate a representation of the cutting surface as a trimmed NURBS surface patch. Its boundary interpolates the given cutting loop, but its interior must not intersect the boundary of the solid.

Two different techniques will be combined in order to solve this problem. First we construct an *implicit spline surface*, that roughly interpolates the cutting loop and stays away from the other parts of the solid's boundary. This surface is obtained using methods for implicit spline surface fitting. In the second part we use techniques for trimmed *spline surface fitting* to obtain a cutting surface that simultaneously approximates the given cutting loop in the boundary and the implicit guiding surface in the interior.

Implicit curves and surfaces are a well-established tool for geometry reconstruction [23–26]. More recently, Wang et al. use implicit PHT-splines to reconstruct curves and surfaces [27], while Pan et al. [28] employ a low-rank tensor approximation technique to reduce the complexity of the required computer memory.

Spline surface fitting addresses the problem of geometry reconstruction using parametric curves and surfaces, see the surveys [29,30] for more information. In particular, techniques for spline surface fitting to implicit surfaces are of interest. Related work includes a paper by Wurm et al. [31], who find a tensor-product spline surface representation of a given algebraic surface by minimizing a non-linear objective function.

The remainder of this article consists of four major parts. First we give a detailed explanation of the cutting problem and introduce the notions that will be used throughout the paper in Section 2. We then formulate a suitable constrained optimization problem in Section 3, which allows us to obtain an implicit guiding surface. As the next step we discuss the construction of a parametric representation of the cutting surface in Section 4. Finally we present several computational results that illustrate our approach in Section 5.

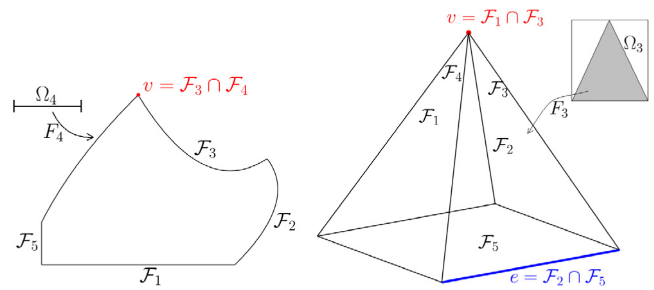

Fig. 2. Left: A 2D solid with five facets (curve segments) and one highlighted ridge (vertex). Right: A 3D solid with five facets (trimmed surface patches) and two highlighted ridges (vertex and edge).

Fig. 1 visualizes the whole procedure: The left two pictures show a three-dimensional solid and a given cutting loop,[1] cf. Section 2. The picture in the center depicts an implicit guiding surface and the last two pictures show the parameterized cutting surface and its automatically generated trimming-loop.

2. Preliminaries

We consider the problem of decomposing a d-dimensional simply connected domain (a solid object), which is given in boundary representation into two smaller simply connected domains for $d = 2, 3$. More precisely we are given a list of n *facets*

$$F_i : \Omega_i \subset [0, 1]^{d-1} \to \mathbb{R}^d \quad \text{for} \quad i = 1, \dots, n,$$

such that $\bigcup_i F_i(\Omega_i)$ is the boundary of a solid object S in $\mathbb{R}^d$. We distinguish between a parameterized facet F_i and its geometric locus $\mathcal{F}_i = F_i(\Omega_i)$. For simplicity we use the notion *facet* for both of them.

In the two-dimensional case ($d = 2$), the parameter domains Ω_i are intervals and the associated facets F_i are simply segments of planar curves. The solid S is the planar domain that is bounded by these segments.

A three-dimensional solid ($d = 3$) is a domain in $\mathbb{R}^3$ that is bounded by surface patches F_i. More precisely, one considers trimmed surface patches, as the parameter domains Ω_i can be general solids in the plane, i.e., planar domains which are bounded by a curve polygon (see Fig. 2).

The non-empty intersections $\mathcal{F}_i \cap \mathcal{F}_j \subset \mathbb{R}^d$ for $i \neq j$ will be called *ridges*. In the planar case ($d = 2$), the ridges are the start- and end-points of the boundary curves and therefore vertices. The ridges can be either edges or vertices for dimension $d = 3$, see Fig. 2.

[1] Note that the cutting loop in this example does not induce a meaningful segmentation of the solid. It was artificially chosen in order to illustrate the required properties of cutting surfaces. This comment also applies to the cutting data in Figs. 5, 6 and 12. In contrast, real cutting loops have been used in Figs. 13 and 15.

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