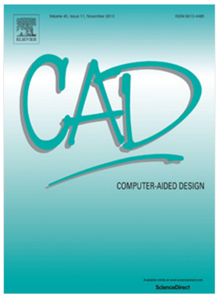

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Skinning and blending with rational envelope surfaces

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Abstract

We continue the study of rational envelope (RE) surfaces. Although these surfaces are parametrized with the help of square roots, when considering an RE patch as the medial surface transform in 4D of a spatial domain it yields a rational parametrization of the domain's boundary, i.e., the envelope of the corresponding 2-parameter family of spheres. We formulate efficient algorithms for G^1 data interpolation using RE surfaces and apply the developed methods to rational skinning and blending of sets of spheres and cones/cylinders, respectively. Our results are demonstrated on several computed examples of skins and blends with rational parametrizations.

Key words: Rational envelope surface, medial axis transform, blending, skinning

1. Introduction

One of the main issues when dealing and computing with geometric objects is the choice of a suitable type of representation [1]. This is important not only from the point of view of geometry representation itself, but also for formulating subsequent and downstream algorithms. A special role is played by parametric representations as these provide easy generation of points on curves and surfaces, and are well suited for surface rendering, computing transformations, determining offsets, admit simple curvature computation, and play a key role in various intersection problems. For these reasons, parametric descriptions are employed in computer graphics and also in computer-aided (geometric) design. Among all parametrizations, the most important ones are those that can be described with the help of (piece-wise) polynomial or rational functions, forming the basis of standard CAD systems as the so-called NURBS objects. The NURBS representation (where NURBS stands for Non-Uniform Rational B-Spline) is considered the universal standard in technical practice, offering a unifying data exchange format and being able to exactly represent, for example, conics, quadrics, and many other elementary geometric objects from technical applications [2], including free-form spline curves and surfaces.

On the other hand, many natural geometric operations applied to NURBS curves or surfaces do not preserve the rationality of the derived objects. Among the most frequent of such operations are offsetting, the operation of convolution, and the construction of envelopes and Minkowski sums. Hence, studying derived or transformed object rationality belongs to challenging problems in geometric modelling [3, 4]. Nonetheless, we consider a more general viewpoint: When the sought-after object should be rational (i.e., representable by a rational parametrization), it is often convenient to use intermediate non-rational representations. The simplest non-rational parametrizations are square-root parametrizations of curves and surfaces. A curve or surface is called square-root parametrizable if it can be rationally parametrized in terms of t or (u, v) and $\sqrt{P(t)}$ or $\sqrt{P(u, v)}$ with polynomial $P(t)$ or $P(u, v)$, respectively.

It is known that any curve given by a square-root parametrization is rational, elliptic, or hyper-elliptic [5]. We exemplify here several constructions based on elliptic or hyper-elliptic curves that lead to rational derived objects. For instance, it has been proved that canal surfaces determined by a rational trajectory of moving spheres and a square-root radius function are rational [6]. Hence, by allowing square-roots in the parametrizations of medial axis transforms, a larger class of rational canal surfaces can be constructed. A similar result holds also for rational ringed surfaces given by a square-root radius function [7]. And recently, the so-called RE curves, i.e., curves yielding rational envelopes, have been introduced [8]. RE curves, although containing square roots, yield rational envelopes and can be constructed by simpler methods than those for Minkowski Pythagorean hodograph curves [9, 10]. They can also be used for canal surface adaptive blending using rational blends.

In this paper we continue the study of [8] and investigate in more detail surface analogies to RE curves. These RE surfaces, considered as medial surface transforms in four-dimensional space [11, 12], are parametrized

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