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Planar multi-patch domain parameterization via patch adjacency graphs

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ABSTRACT

As a remarkable difference to the existing CAD technology, where shapes are represented by their boundaries, FEM-based isogeometric analysis typically needs a parameterization of the interior of the domain. Due to the strong influence on the accuracy of the analysis, methods for constructing a good parameterization are fundamentally important. The flexibility of single patch representations is often insufficient, especially when more complex geometric shapes have to be represented. Using a multi-patch structure may help to overcome this challenge.

In this paper we present a systematic method for exploring the different possible parameterizations of a planar domain by collections of quadrilateral patches. Given a domain, which is represented by a certain number of boundary curves, our aim is to find the optimal multi-patch parameterization with respect to an objective function that captures the parameterization quality. The optimization considers both the location of the control points and the layout of the multi-patch structure. The latter information is captured by pre-computed catalogs of all available multi-patch topologies. Several numerical examples demonstrate the performance of the method.

1. Introduction

The powerful framework of Isogeometric Analysis (IgA) unifies the descriptions of the geometric objects (the computational domains) and of the unknown quantities considered in numerical simulations by adopting multivariate spline functions for their representation [1]. This approach helps to deal with the problem of data exchange between the software tools used for the geometrical construction and design of CAD models and the tools used for analysis. Additionally, the higher smoothness of the spline functions has shown to be beneficial for the robustness and accuracy of the numerical simulations.

In the existing CAD technology, shapes are represented by boundary representation (B-rep) models, which contain information about the surfaces of geometric objects [2]. However, IgA typically needs a parameterization of the interior of the domain. Domain parameterization techniques, which generate domain parameterizations from boundary data, are therefore of a vital interest.

The starting point for exploring such methods is the case of a single patch, i.e., of a domain that is topologically equivalent to a d-dimensional cube of dimension $d = 2, 3$. Several techniques have been described in the literature.

These include discrete Coons patches [3], which are computationally inexpensive, or the spring model based approach [4]. A direct construction of spline parameterizations for swept volumes was established in [5]. A method for generating volumetric single-patch T-spline parameterizations of contractible objects is described in [6]. Optimal analysis-aware parameterizations were studied in [7].

For two-dimensional domains, a sequence of methods with varying levels of computational complexity has been presented in [8]. The first two methods make use of several functionals to place the inner control points, while the third method employs a harmonic mapping to guarantee regularity. Harmonic functions are used in [9,10] also to construct parameterizations of generalized cylinders and of contractible domains, respectively. Another method, that is specifically tailored towards single-patch NURBS-parameterization, has been developed in [11] using sequences of harmonic mappings.

When dealing with more complex geometric shapes, however, single patch representations do not provide sufficient flexibility.

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The use of multi-patch structures, which are obtained by gluing patches together, is a standard approach to make them more versatile.

The coupling of the patches can be performed in several ways. One may identify degrees of freedom along the interfaces. This has led to special constructions for splines on multi-patch domains, such as spline forests [12] or multi-patch B-splines with enhanced smoothness [13]. Alternatively, one may use continuity constraints and enforce them via Lagrangian multipliers. This was found to be useful for designing isogeometric solvers that exploit the power of parallel computing [14,15]. Finally, there exist numerous techniques such as use of mortar methods, Nitsche's method, and the discontinuous Galerkin method [16–18].

The construction of multi-patch parameterizations in IgA is less well understood. For a fixed topology, an optimization-based technique has been investigated in [19]. Skeleton-based polycube constructions have been used to generate T-spline parameterizations of multi-patch-type [20]. An isogeometric segmentation pipeline, which combines domain splitting with single patch parameterization techniques, was established in [21–23]. It should be noted that there are strong relations to the problem of block-structured hexahedral mesh-generation, see [24] and the references therein. Recently, a systematic method for exploring the possible quad mesh topologies has been presented in [25].

In our present paper, which focuses on the two-dimensional case, we establish a systematic method for exploring the different possible segmentations of a domain into quadrilateral patches. Given a domain with a certain number of boundary curves, which we denote as segments, our aim is to find the optimal multi-patch parameterization with respect to an objective function that captures the parameterization quality. The optimization considers both the location of the control points and the layout of the multi-patch structure. This is achieved by using catalogs of all available multi-patch topologies.

This paper is organized as follows. First we recall the concept of a multi-patch parameterization and introduce the patch adjacency graph in Section 2. The outline of the algorithm as well as the objective functions used for the geometry optimization are presented in the next section. Finally we demonstrated the performance of the proposed method by several computational examples in Section 4. The method used for enumerating the multi-patch topologies is reported in the Appendix.

2. Preliminaries

We introduce the notion of a multi-patch parameterization and show how to capture its topology by a patch adjacency graph.

2.1. Valid multi-patch parameterizations

We consider a simply connected planar domain Ω with piecewise smooth boundary. More precisely, the boundary is represented by b parametric curves,

$$c_i : [0, 1] \to \mathbb{R}^2, \quad i = 1, \ldots, b, \tag{1}$$

which are called *segments*. Note that the indices of these curves are always considered modulo b. The segments do not intersect except for the end points of neighboring segments,

$$\forall i : c_i(1) = c_{i+1}(0) \quad \text{and}$$
$$\forall i, j,\ \forall s, t \in [0, 1) : c_i(s) = c_j(t) \Rightarrow (i = j \wedge s = t).$$

The first property ensures that the segments form a closed loop. Without loss of generality, we assume that the segments are oriented clockwise.

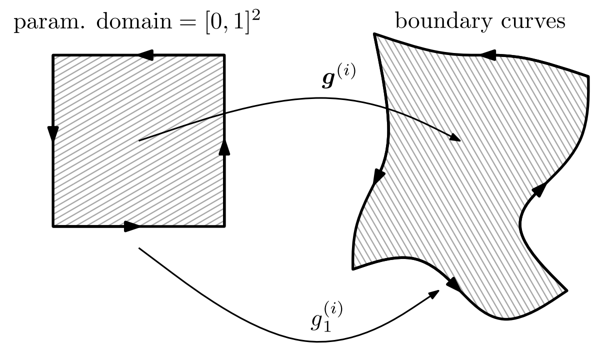

Fig. 1. Parameter domain and loop of boundary curves for a patch i.

Some pairs of adjacent segments may meet with tangent (G^1) continuity, i.e.,

$$\exists \lambda > 0 : \dot{c}_i(1) = \lambda \dot{c}_{i+1}(0).$$

Adjacent edges meet at *corners*, which are either *convex* if the inner angle is smaller than π or *non-convex* otherwise. The latter notion also includes the case of tangent continuity between adjacent segments.

We want to represent the domain, i.e., the interior of the loop formed by the segments, by a *multi-patch parameterization* (MPP). More precisely, the parameterization consists of p geometry patches $\mathbf{g}^{(j)}$.

$$\mathbf{g}^{(j)} : [0, 1]^2 \to \mathbb{R}^2 \quad j = 1, \ldots, p.$$

A patch is *regular* if the determinant of the Jacobian $\nabla \mathbf{g}^{(j)}$ is positive. In particular, this implies that the corner angles do not exceed π. Every patch $\mathbf{g}^{(j)}$ has a loop of four boundary curves

$$g_1^{(i)} : [0, 1] \to \mathbb{R}^2 : t \mapsto \mathbf{g}^{(i)}(0, t)$$
$$g_2^{(i)} : [0, 1] \to \mathbb{R}^2 : t \mapsto \mathbf{g}^{(i)}(t, 1)$$
$$g_3^{(i)} : [0, 1] \to \mathbb{R}^2 : t \mapsto \mathbf{g}^{(i)}(1, 1 - t)$$
$$g_4^{(i)} : [0, 1] \to \mathbb{R}^2 : t \mapsto \mathbf{g}^{(i)}(1 - t, 0),$$

with counterclockwise orientation, see Fig. 1.

We use the opposite (i.e., clockwise) orientation for the boundary of the domain in order to prepare the definition of the adjacency relation (see below), which identifies pairs of curves with opposite orientation.

All boundary curves and the segments forming the domain boundary are collected into a set

$$S = \{g_\ell^{(i)} \mid \ell = 1, \ldots, 4;\ i = 1, \ldots, p\} \cup \{c_i \mid i = 0, \ldots, b - 1\}.$$

The interface relation $\sim$ on S identifies curves that possess the same parameterization in reverse orientation,

$$s \sim s' \Leftrightarrow s(t) = s'(1 - t), \quad \forall t \in [0, 1]\, s, s' \in S.$$

It captures both the adjacency relation between patches and between a patch and the domain boundary. This relation is symmetric but neither reflexive nor transitive, and it does not relate pairs of segments c_i and c_j.

A set of patches $\{\mathbf{g}^{(j)}\}$ is called a *valid* multi-patch parameterization if it satisfies the following three conditions:

(P) All patches are regular and their interiors are mutually disjoint.
(R1) Every curve in S, which can be either a segment (on the domain boundary) or a patch boundary curve, is related to exactly one other curve in S.
(R2) There are no relations between the boundary curves of each patch.

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