Gauss-Galerkin quadrature rules for quadratic and cubic spline spaces and their application to isogeometric analysis

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Abstract

We introduce Gaussian quadrature rules for spline spaces that are frequently used in Galerkin discretizations to build mass and stiffness matrices. By definition, these spaces are of even degrees. The optimal quadrature rules we recently derived [6] act on spaces of the smallest odd degrees and, therefore, are still slightly sub-optimal. In this work, we derive optimal rules directly for even-degree spaces and therefore further improve our recent result. We use optimal quadrature rules for spaces over two elements as elementary building blocks and use recursively the homotopy continuation concept described in [5] to derive optimal rules for arbitrary admissible numbers of elements. We demonstrate the proposed methodology on relevant examples, where we derive optimal rules for various even-degree spline spaces. We also discuss convergence of our rules to their asymptotic counterparts, these are the analogues of the midpoint rule of Hughes et al. [18], that are exact and optimal for infinite domains.

Keywords: optimal quadrature rules, Galerkin method, Gaussian quadrature, B-splines, isogeometric analysis, homotopy continuation for quadrature

1. Introduction and motivation

Numerical integration is a fundamental ingredient of isogeometric analysis (IGA) and finite elements (FE), and its computational efficiency is essential. When simulating physical processes, e.g., [10, 11, 16, 20, 32], with Galerkin

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