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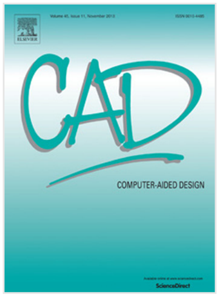

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Splines over regular triangulations in numerical simulation

Francesca Pelosi[a], Carlotta Giannelli[b], Carla Manni[a], Maria Lucia Sampoli[c], Hendrik Speleers[a]

[a] *Department of Mathematics, University of Rome "Tor Vergata", Italy*
[b] *Department of Mathematics and Computer Science, University of Florence, Italy*
[c] *Department of Information Engineering and Mathematics, University of Siena, Italy*

Abstract

We investigate the use of smooth spline spaces over regular triangulations as a tool in (isogeometric) Galerkin methods. In particular, we focus on box splines over three-directional meshes. Box splines are multivariate generalizations of univariate cardinal B-splines sharing the same properties. Tensor-product B-splines with uniform knots are a special case of box splines. The use of box splines over three-directional meshes has several advantages compared with tensor-product B-splines, including enhanced flexibility in the treatment of the geometry and stiffness matrices with stronger sparsity. Boundary conditions are imposed in a weak form to avoid the construction of special boundary functions. We illustrate the effectiveness of the approach by means of a selection of numerical examples.

Keywords: Isogeometric analysis; Galerkin methods; box splines; three-directional meshes; weak boundary conditions.

1. Introduction

Isogeometric Analysis (IgA) is a technology created about ten years ago with the aim to bridge the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). The key concept in IgA is the development of a new isoparametric paradigm for FEA where the same basis functions used for geometry representations in CAD systems are adopted for the approximation of field variables [6, 13]. The isogeometric framework results in some important advantages. In particular, it gives more flexibility to generate and to refine the computational mesh, it provides a more accurate description of the geometry, and it allows an easy treatment and refinement of spaces with high approximation order and high smoothness, leading to a higher accuracy per degree-of-freedom. IgA has been successfully applied in various areas (see, e.g., [6, 14] and references therein), and it is rapidly becoming a mainstream analysis methodology and a new paradigm for geometric design [22].

In its original formulation, IgA is based on tensor-product B-splines and their rational version NURBS. The main advantages of tensor-product B-splines/NURBS are their easy implementation and computational efficiency, regardless of the dimensionality of the problem. Unfortunately, the tensor-product structure has also few drawbacks. First of all, (single-patch) NURBS lack an adequate local refinement. Moreover, they fail a proper modeling of complex geometries and require the use of singular geometry maps to represent physical domains having no "box-like shape". These are, in the bivariate setting for example, domains having a number of corners different from four. Yet, the quality of the parameterization plays a crucial role in IgA, because it has a significant impact on the simulation results and the efficiency of the computations, see [21, 26, 32] and references therein.

Nevertheless, the advantages of IgA are not a distinguishing feature of tensor-product B-splines/NURBS. A powerful alternative is the use of splines over triangulations in IgA. This offers a natural solution to efficient local refinement strategies [2, 15, 28, 29, 31] as well as to high-quality parameterizations [26]. However, the full flexibility of smooth spline spaces over general triangulations comes at a certain price: it requires quite involved implementations when no suitable (B-spline-like) bases are considered, and the treatment of high-dimensional problems can be quite cumbersome both from the theoretical and computational point of view.

As an intermediate step between the simplicity of tensor-product splines and the richness and flexibility of splines over general triangulations, in this paper we investigate the use of spline spaces over regular triangulations as a tool in numerical simulation. In particular, we focus on box splines over three-directional meshes in the frame of (isogeometric) Galerkin methods.

Box splines are multivariate generalizations of univariate cardinal B-splines, i.e. B-splines with uniform knots, see [7, 4, 8, 20]. They enjoy the same properties as (cardinal) B-splines: piecewise polynomial structure, compact support, positivity, recurrence relation and convolution properties. A d-variate box spline is determined by (possibly

Email addresses: pelosi@mat.uniroma2.it (Francesca Pelosi), carlotta.giannelli@unifi.it (Carlotta Giannelli), manni@mat.uniroma2.it (Carla Manni), marialucia.sampoli@unisi.it (Maria Lucia Sampoli), speleers@mat.uniroma2.it (Hendrik Speleers)