ARTICLE IN PRESS COMAID:1604

Computer Aided Geometric Design ••• (••••) •••–•••

Contents lists available at ScienceDirect

Computer Aided Geometric Design

www.elsevier.com/locate/cagd

Towards optimal advection using stretch-maximizing stream surfaces

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Stream surface
Seed curve
Advection
Flow visualization

ABSTRACT

We investigate a class of stream surfaces that expand in time as much as possible. Given a vector field, we look for seed curves that locally propagate in time in a stretch-maximizing manner, i.e., curves that infinitesimally expand most progressively. We show that such a curve is generically unique at every point in an incompressible flow and offers a very good initial guess for a stretch-maximizing stream surface. With the application of efficient fluid advection–diffusion in mind, we optimize fluid injection towards optimal advection and show several examples on benchmark datasets.

1. Introduction

We investigate a special class of stream surfaces generated by seed curves that maximize a certain arc-length energy. We aim at an efficient advection process where one typically requires to minimize the cost of injection while speeding up (or maximizing) the advection effect. To achieve that, we focus on globally optimizing the geometric advection of the injected seed curve since this is closely related to seed curves that expand in time with the maximum stretching rate. The corresponding stream surface exhibits the maximum possible advection of the injected fluid (Hundsdorfer and Verwer, 2013).

Recent research in this area has been devoted to oil extraction from porous media by gas diffusion in both fractured and unfractured reservoirs, see e.g. Hoteit et al. (2009) and the references cited therein. The efficiency in terms of the time required as well as the amount of solvent injected into a porous medium was studied in Trivedi and Babadagli (2008). Another possible application of our research points to efficient crop spraying (Kamal et al., 2014), where the trajectory of an aircraft is optimized to maximize the area covered by the sprayed pesticide.

With these applications in mind, we seek injection (or seed) curves (McLoughlin et al., 2010) that propagate in time in the most progressive manner, by stretching their lengths as much as possible. For a steady state vector field, our goal is to find a seed curve such that the corresponding stream surface facilitates efficient advection.

We extend previous work on stretch-minimizing stream surfaces (Bartoň et al., 2015) where a different special class of stream surface was investigated. Those surfaces possess the property that their seed curves propagate in time such that their arc-lengths are as constant as possible, and therefore help to detect parts of the domain where the given divergence-free vector field acts not only in a volume-preserving but also in a length-preserving manner. In contrast, in this work, we seek stream surfaces arising from seed curves that advect in time with the maximum stretching rate, see Fig. 1, left.

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http://dx.doi.org/10.1016/j.cagd.2017.02.010
0167-8396/

Please cite this article in press as: Bartoň, M., Kosinka, J. Towards optimal advection using stretch-maximizing stream surfaces. Comput. Aided Geom. Des. (2017), http://dx.doi.org/10.1016/j.cagd.2017.02.010

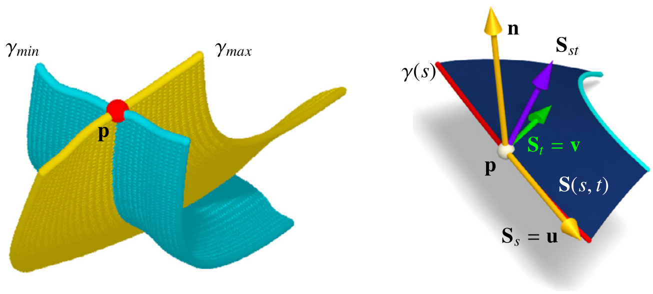

Fig. 1. Left: Let $\mathbf{p}$ be a seed point in a vector field $\mathbf{v}$. A stretch-minimizing curve γ_{min} advects infinitesimally in $\mathbf{v}$ with a zero stretching rate, while a stretch-maximizing curve γ_{max} stretches with the maximum possible rate and consequently maximum local advection. Right: Differential geometry of stream surfaces. The stream surface $\mathbf{S}(s,t)$ (dark blue) is generated by the seed curve $\gamma(s)$ (red) by integrating it through $\mathbf{v}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Problem formulation. Given a vector field $\mathbf{v}$ in some domain $\Omega \subset \mathbb{R}^3$, find a *stream surface* such that the *seed curve* that defines it propagates in time such that its arc-length expands as much as possible.

We combine theoretical results and a practical algorithm for finding such surfaces. The main contributions of our method are:

- We theoretically investigate families of seed curves based on a certain stretch-maximizing energy (Section 3).
- These candidate seed curves are used to initialize stream surfaces, which are subsequently globally optimized (Section 4).

We provide implementation details of our method in Section 5. Our results are presented and discussed in Section 6. We include standard benchmark datasets and we also validate our algorithm on an analytic vector field with known stretch-maximizing stream surface solutions. The paper is concluded in Section 7.

2. Related work

Vector fields are widely used in geometry processing for object propagation and deformation (Hildebrandt et al., 2005; von Wolfram et al., 2006; Bartoň et al., 2013). For example for deformation purposes, appropriate vector fields are iteratively sought-after such that an object under consideration (curve, solid) preserves its certain measure (arc-length, volume). Visualization of vector fields is a very active area; see Bauer and Peikert (2002), Lentine et al. (2010), Edmunds et al. (2012b), Edmunds et al. (2012c), and the survey paper McLoughlin et al. (2010) and the references cited therein. Among flow visualization techniques, stream surfaces play an important role (von Funck et al., 2008; Schulze et al., 2014; Schneider et al., 2010; Martinez Esturo et al., 2013; Edmunds et al., 2011; Edmunds et al., 2012a).

Given a vector field, the main goal is to select a seed curve such that its stream surface captures well the characteristic features of the field. While classical results like Hultquist (1992) admit user's intervention to set seed curves in a trial-and-error manner, recent research focuses on fully *automatic* stream surface seeding (Edmunds et al., 2011; Edmunds et al., 2012a; Bartoň et al., 2015). Our work belongs to this category of automatic methods.

As shown in Fig. 1, our work on optimal advection can be seen as a modification of the concept of stretch-minimizing stream surfaces (Bartoň et al., 2015). Our framework differs from that of Bartoň et al. (2015) in several aspects, which require a separate treatment as detailed below.

Our problem formulation is closely related to the concept of Lyapunov stability. In fact, the sought-after seed curves could be seen as loci of points where the corresponding dynamical system (represented by the input vector field) is as unstable as possible. This is formalized by the notion of Lyapunov exponents (Peikert et al., 2014; Waldner and Klages, 2012). In our approach, we employ the local variant of the Lyapunov exponent (Eckhardt and Yao, 1993).

3. Stretch-energy maximizing curves

Let $\mathbf{v}(\mathbf{p})$ be a steady differentiable vector field defined over a domain $\Omega \subset \mathbb{R}^3$. Let $\mathbf{J}$ be the Jacobian matrix of $\mathbf{v}(\mathbf{p})$, i.e., $\mathbf{J}_{ij} = \frac{\partial \mathbf{v}_i}{\partial p_j}$ with $\mathbf{p} = (p_1, p_2, p_3)$. Consider a regular *seed curve* $\gamma(s)$ parametrized by arc-length, $s \in [s_0, s_1]$, that gives rise to a *stream surface* $\mathbf{S}(s,t)$ defined on $[s_0, s_1] \times [t_0, t_1]$, with $\mathbf{S}(s,0) = \gamma(s)$; see Fig. 1, right. The partial derivatives of $\mathbf{S}$ will be denoted $\mathbf{S}_s$, $\mathbf{S}_{st}$, etc.

The Taylor expansion of the arc-length of the timelines of $\mathbf{S}(s,t)$ with respect to t at $t = 0$ (which corresponds to γ) is given by

$$\int_{s_0}^{s_1} ||\mathbf{S}_s(s,t)||\, \mathrm{d}s = (s_1 - s_0) + ct + \mathcal{O}(t^2) \tag{1}$$

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