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Surface reconstruction using simplex splines on feature-sensitive configurations ☆

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ABSTRACT

Recently, new non-tensor product splines, called as triangle configuration B-splines (TCB-splines), have been proposed, which generalize the Delaunay configuration B-splines (DCB-splines) by introducing more flexibility in knot geometry while preserving the most attractive properties of univariate B-splines (Liu and Snoeyink, 2007). In this paper, we present a surface reconstruction framework that explores the flexibility of this spline for its use in shape modeling. Starting from a carefully designed feature-sensitive triangulation, we apply the so-call link triangulation procedure to obtain a feature-sensitive configuration family and then construct simplex spline bases as well as spline surface on it. The approximation quality of the surface is then progressively improved by adaptively adding more knots onto the parametric domain according to the fitting errors obtained in the previous fit procedure and updating the feature-sensitive configurations. Conditions to ensure the C^1 continuity of the whole result surface are also provided. Our framework is tested on several models to demonstrate its efficacy and ability in preserving geometric features. Compared with DCB-splines, our splines can generate visually pleasant surfaces with smaller fitting errors by using the same number of knots and iterations. With almost the same number of control points, our framework produces more accurate and visually pleasant results than the classical B-spline surface fitting method based on adaptive knot placement strategy (Park, 2011). In addition, the resultant surface provides a control net, which enables an intuitive user interactive design.

1. Introduction

Surface reconstruction is an indispensable procedure of converting scattered data into continuous and compact expressions in the fields of computer graphics, reverse engineering, and computer-aided geometric design, among others. In surface reconstruction, three kinds of mathematical tools are mainly used for surface representation, that is, implicit surface (Zhao et al., 2001), subdivision surface (Hoppe et al., 1994), and parametric spline surface (Farin et al., 2002). Among these tools, parametric spline possesses a number of properties, which make it particularly attractive for use in the area of data fitting, geometric modeling, numerical solution of differential equations, and diverse downstream analysis.

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In existing parametric splines, B-spline (including the special case Bézier) and its rational form NURBS (non-uniform rational B-spline) enjoy immense popularity owing to their attractive properties, such as geometrical intuition, robust computation, local control, convex hull, optimal smoothness. However, B-spline surface, as obtained from univariate B-splines by tensor-product method, requires a regular knot topology (on a grid pattern), which means that it can not be easily used to model a wide variety of objects. Specifically, complicated shapes cannot be satisfactorily modeled unless multiple patches are stitched together or holes are filled with patches of different types, which tend to circumvent the desirable qualities. Therefore, many studies have proposed new parametric splines that preserve the desirable properties of B-splines while allowing more general knot topology.

1.1. Generalization of B-spline surface

B-spline surfaces are basically generalized in two manners: tensor product generalization and non-tensor product generalization. There are a plenty of research works on generalizing B-spline in tensor product way and developing their application algorithms (see Eck and Hoppe, 1996; Lee et al., 1997; Gregorski et al., 2000; Floater and Reimers, 2001; Xie et al., 2004; Sederberg et al., 2003, 2004; He et al., 2006; Deng et al., 2008; Scott et al., 2012; Li et al., 2007; Bazilevs et al., 2010, and the references therein). A trivial generalization of B-splines is the tensor products of other univariate splines, such as NUAT B-splines (Wang et al., 2004). Perhaps the most successful approach of tensor-product generalization of B-splines is the T-spline, which advances the B-splines by constructing basis on knot grids with junctions, i.e., the lines of knots do not need to traverse the entire parametric domain (Sederberg et al., 2003, 2004). T-splines themselves are also generalized in many ways (He et al., 2006; Deng et al., 2008; Scott et al., 2012) and applied in different applications (Li et al., 2007; Bazilevs et al., 2010). Generally speaking, tensor product generalization focuses on constructing basis on regular or semi-regular knot geometries. As a result of their intrinsic rectangular shapes, they cannot easily be used to model shapes with even a simple topology while retaining desired properties, such as a specific continuity.

Non-tensor product generalization constructs basis on irregular knot geometries, including triangle domain (Farin, 1986), polygonal domain (Gregory, 1986), triangulated knots (Lai and Schumaker, 2007; Dahmen et al., 1992), regular triangular mesh (de Boor et al., 1993), or scattered knots (Prautzsch et al., 2002; Neamtu, 2001b; Feng and Warren, 2012). Compared with tensor product type splines, non-tensor product splines are less popular in fields related to shape modeling, mainly because of their computational difficulties or low approximation quality. In fact, non-tensor product splines have more potential in modeling complicated shapes as they usually allow more general knot geometries. Of these non-tensor product splines, simplex spline, as a single basis, has much in common with the univariate B-spline. In particular, the underlying knots can be chosen almost arbitrarily while preserving optimal smoothness. However, arbitrary knot configurations may not produce a simplex spline space with approximation properties familiar from the univariate B-spline. Subsequently, many researchers focus on finding an appropriate choice of knot configurations that generates simplex spline space with desired properties. The well-known triangular B-spline (DMS-spline) is a classical example of such spline spaces. Many studies derive DMS-spline-based algorithms to reconstruct surface or solve PDEs (see, for example, He and Qin, 2004; He et al., 2005 and Jia et al., 2013). However, DMS-spline requires deliberate attachment of each original knot with some auxiliary knots, which is apt to introduce “knot-line” phenomena on the resultant surface. As a result, the surface is visually unpleasant in the region that corresponds to the segments that pass two knots on the parametric domain.

Delaunay configuration B-spline or DCB-spline, which uses circles to enclose knot sets as knot configurations, is also a simplex spline-based spline space (Neamtu, 2001a, 2001b, 2007). The construction of DCB-splines was rigorously proved in Neamtu (2007). It is said to be the most successful multivariate generalization of univariate B-splines (de Boor, 2009). Subsequently, many studies have focused on developing the theory (Liu and Snoeyink, 2007), the application algorithms on shape modeling (Cao et al., 2009, 2012), data fitting and collocation (Dembart et al., 2005), or image registration (Hansen et al., 2008). In particular, DCB-spline theory is augmented with a more general knot configuration selection method in Liu and Snoeyink (2007), where the Delaunay configuration becomes a special case. The splines proposed by Liu and Snoeyink (2007) maintain all the attractive properties of DCB-spline, while providing more flexibility for modeling or other applications. This theoretical advantage inspires us to explore its usage in practical applications.

1.2. Contributions

The simplex spline space proposed by Liu and Snoeyink (2007) is constructed on so-called triangle configurations, with the associated splines called triangle configurations B-splines (TCB-splines). In this study, we present some first steps towards the practical application of this spline. In particular, we develop a TCB-spline-based surface reconstruction framework that iteratively improves surface quality by exploring the flexibility in TCB-splines to model feature regions. For DCB-splines, the knot configurations are determined once the knots are placed. Unlike that approach, our framework first attempts to select the knot configurations adaptively according to the input data’s geometric features. Hence, we provide additional flexibility for modeling geometric details. The reconstructed surface also provides a control net for the interactive shape design, which is superior to the case of DCB-splines with an ambiguous control net. Compared with the DMS-spline-based surface reconstruction method, the proposed method eliminates the requirement of adding auxiliary knots, whose influence on the final surface is unpredictable. Moreover, our method generates more accurate and visually pleasant results than DOM-based B-spline does (Park, 2011) with almost the same number of control points.