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Logarithmic dyadic wavelet transform with its applications in edge detection and reconstruction

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ABSTRACT

In this paper, based on the logarithmic image processing model and the dyadic wavelet transform (DWT), we introduce a logarithmic DWT (LDWT) that is a mathematical transform. It can be used in image edge detection, signal and image reconstruction. Comparative study of this proposed LDWT-based method is done with the edge detection Canny and Sobel methods using Pratt's Figure of Merit, and the comparative results show that the LDWT-based method is better and more robust in detecting low contrast edges than the other two methods. The gradient maps of images are detected by using the DWT- and LDWT-based methods, and the experimental results demonstrate that the gradient maps obtained by the LDWT-based method are more adequate and precisely located. Finally, we use the DWT- and LDWT-based methods to reconstruct one-dimensional signals and two-dimensional images, and the reconstruction results show that the LDWT-based reconstruction method is more effective.

1. Introduction

The logarithmic image processing (LIP) model is a technique initially stated by Jourlin and Pinoli [1] and further developed in [2]. The parametrized LIP model [3] is a generalization of the LIP model which attempts to overcome the mentioned shortcomings of the standard processing. The LIP theory is a mathematical framework that provides a set of specific algebraic and functional operations and structures that are well adapted to the representation and processing of non-linear images, valued in a bounded intensity range [4]. It has been used in many image processing tasks such as edge detection and image enhancement. Recently, many researchers have developed effective edge detection algorithms based on LIP such as in [5–10].

Since the dyadic wavelet transform (DWT) is introduced in [11], it has been widely used in image edge detection (e.g. [12–15]), signal and image reconstruction (e.g. [11,16]), etc. In image processing, the traditional edge detection methods can be grouped into two categories: gradient method (e.g. Sobel, Roberts, Prewitt) and zero-crossing method (e.g. Laplacian of Gaussian, zero-cross). The gradient method detects the edges by looking for the maximum and minimum in the first derivative of the image, and the zero-crossing method searches for zero crossings in the second derivative of the image to find edges. However, these gradient-based and zero-crossing finding algorithms are very sensitive to noise. Thus, a better edge detection method, Canny edge detector [17] occurs. It improves the resistance of the noise and the scale of the Gaussian filter is able to be adjusted. However, the Canny operator still suffers from some practical limitations (e.g., the performance of the Canny algorithm relies mainly on the changing parameters). The DWT has the good locality and multi-scale identity, and it is recognized as an efficient way to detect edges and equivalent to the Canny edge detector. It satisfies the need of edge detection in multi-scales so that the noise can be avoided, more real edges will be kept, and the practical limitation of Canny can be overcome.

Many recent studies have shown that the wavelet transform is a good method for edge detection such as in [18–22]. For exapmle, the authors in [18] proposed a scale multiplication based edge detection scheme in wavelet domain. The wavelet transforms at two adjacent scales are multiplied to magnify the edge structures and suppress the noise. The method determines the edges as the local maxima directly in the scale product after an efficient thresholding, and achieves better results on the localization performance in natural images. Unfortunately, it requires a properly determining threshold to suppress the noise maxima. In [23], the authors reviewed wide range of methods of edge detection for image segmentation and concluded that wavelets based methods

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are more accurate than other methods, i.e., the wavelet-based edge detection scheme is considered better than the traditional methods and the Canny operator.

In this paper, we introduce a logarithmic DWT (LDWT), which is an integration of the LIP model and the DWT. In the LIP model, an edge can be defined a positive graytone function within the gray range that corresponds to an intensity of image, and this model associated to the wavelet transform emphasizes certain features such as transitions in a specific range of gray level that are not enhanced using wavelet transform [24]. Thus, the certain features can be used to edge detection and reconstruction.

As known, the wavelet transform modulus maxima (WTMM) representation [11] is a multiscale contour representation of an image. This representation is obtained by retaining the local modulus maxima of the DWT which correspond to discontinuities in the image. Based on the WTMM, some image reconstruction algorithms are proposed in the literature such as in [16,25]. Similarly, we propose a logarithmic WTMM (LWTMM) representation, and a reconstruction algorithm by using the LWTMM.

This paper is organized as follows: Section 2 provides a brief of overview the theory that includes the LIP model and the DWT. Section 3 defines the LDWT and describes the proposed methods. Section 4 presents the experimental results. Section 5 draws discussion based on the experimental results.

2. Methods

In this section, we describe the LIP model and the DWT. The symbols 1D and 2D stand for one- and two-dimensional, respectively.

2.1. Logarithmic image processing

A key point in using LIP model is to carefully distinguish between variables in the two domains: the graytone and the gray level. The relationship between a graytone function f and its corresponding classical gray level function $\tilde{f}$ is defined by $f = M - \tilde{f}$, where $M \in R$ (i.e. real number). The value of a graytone function at a spatial location is called a graytone, and the real number range interval $[0, M)$ is thus called the gray tone range. In order to be able to work in an algebraic vector space, the range has been extended to $f \in (-\infty, M)$. In this paper, the graytone operators can be defined in the set of graytone $E = (-\infty, M)$.

The two following operations, addition $\oplus$ and scalar multiplication $\otimes$ on set E [26,27], can establish a real vector space structure $(\mathbb{E}, \oplus, \otimes)$:

- The addition $\oplus$ between two graytones is defined as:

$$\forall u_1, u_2 \in E, \quad u_1 \oplus u_2 = \frac{u_1 + u_2}{1 + \frac{u_1 \cdot u_2}{M^2}}. \tag{1}$$

- The scalar multiplication $\otimes$ of a graytone u with a real scalar $\alpha \in R$ is:

$$\alpha \otimes u = M \cdot \frac{(M+u)^\alpha - (M-u)^\alpha}{(M+u)^\alpha + (M-u)^\alpha}. \tag{2}$$

In the LIP model, the $(\mathbb{E}, \oplus, \otimes)$ is algebraically isomorphic to the real number space $(\mathbb{R}, +, \times)$ by the mapping $\Psi : \mathbb{E} \longrightarrow \mathbb{R}$:

$$\forall u \in \mathbb{E}, \quad \Psi(u) = -M \cdot \ln(1 - \frac{u}{M}), \tag{3}$$

where *ln* is the natural logarithm. Thus, the isomorphic transform of a graytone u is denoted by $\tilde{u} = \Psi(u)$, where $\tilde{u}$ is a real number. For example, it is easy to show that $\Psi(u_1 \oplus u_2) = \tilde{u}_1 + \tilde{u}_2$ and $\Psi(\alpha \otimes u) = \alpha\tilde{u}$. The inverse isomorphic transformation is then defined as:

$$\Psi^{-1}(\tilde{u}) = M \cdot [1 - \exp(\frac{-\tilde{u}}{M})], \tag{4}$$

where *exp* is the exponential function. We also extend the following operators:

- The graytone multiplication $\odot$ can be defined by

$$\forall u_1, u_2 \in \mathbb{E}, \quad u_1 \odot u_2 = \Psi^{-1}(\Psi(u_1) \cdot \Psi(u_2)). \tag{5}$$

- The standard convolution operator $*$ is used in the DWT. Therefore, a graytone convolution $\circledast$is defined:

$$\forall u \in \mathbb{E}, \quad (u \circledast w) = \Psi^{-1}(\Psi(u) * w), \tag{6}$$

  where w is a filter (e.g. a Gaussian filter), which is convoluted with $\Psi(u)$ that belongs to the real number space $\mathbb{R}$.
- The LIP summation can be defined as

$$\forall u_i \in \mathbb{E}, \quad \sum_{i=1}^{n} \oplus u_i = u_1 \oplus u_2 \oplus \cdots \oplus u_n. \tag{7}$$

It is important to note that the graytone space $\mathbb{E}$ is totally isomorphic to the real number space $\mathbb{R}$ in which it preserves the algebraic, topological and order structures [4], i.e., the manipulation of gray tone operations is equivalent to the manipulation of their corresponding isomorphic transforms with the usual operations.

2.2. Dyadic wavelet transform

The discrete DWT of a signal is implemented using halfband lowpass and highpass filters forming a filterbank together with downsamplers. The filterbank produces two sets of coefficients: orthogonal detail (or wavelet) coefficients which are the even outputs of the highpass filter, and the coarse (or approximation) coefficients which are the even outputs of the lowpass filter.

The 1D wavelet transform of a function $\tilde{f}(x)$ (i.e. $\tilde{f} \in L^2(\mathbb{R})$) at scale 2^j is defined as $\{\mathcal{W}_{2^j}\tilde{f}\}_{j\in\mathbb{Z}}$ with $\mathcal{W}_{2^j}\tilde{f} := \tilde{f} * \psi_{2^j}$, where $\psi_{2^j}(x) = (1/2^j)\psi(x/2^j)$ is a wavelet ψ expanded by a dilation parameter 2^j. $\mathcal{W}_{2^j}\tilde{f}$ is referred to as the detail component. Let $\phi(x)$ be a scaling function, at each scale 2^j, the related scaling coefficients are computed by $\mathcal{S}_{2^j}\tilde{f} := \tilde{f} * \phi_{2^j}$, which is referred to as the coarse component, and $\phi_{2^j}(x) = (1/2^j)\phi(x/2^j)$ is ϕ expanded by a dilation parameter 2^j. For a J-level, the collection $\{\mathcal{S}_{2^J}\tilde{f},\ \{\mathcal{W}_{2^j}\tilde{f}\}\}_{1\le j\le J}$ is called the 1D discrete DWT.

For an image $\tilde{f}(x, y)$ (i.e. $\tilde{f} \in L^2(\mathbb{R}^2)$), the 2D wavelet transform of $\tilde{f}$ at scale 2^j is defined as $\mathcal{W}^1_{2^j}\tilde{f} := \tilde{f} * \psi^1_{2^j}$ and $\mathcal{W}^2_{2^j}\tilde{f} := \tilde{f} * \psi^2_{2^j}$, where $\psi^1_{2^j}(x, y) = (1/2^j)\psi^1(x/2^j, y/2^j)$ and $\psi^2_{2^j}(x, y) = (1/2^j)\psi^2(x/2^j, y/2^j)$ are the wavelets $\psi^1(x,\ y)$ and $\psi^2(x,\ y)$ expanded by a dilation parameter 2^j, respectively. As 1D, let $\phi(x, y)$ be a real function. The coarse component of the image $\tilde{f}$ at scale 2^j is $\mathcal{S}_{2^j}\tilde{f} := \tilde{f} * \phi_{2^j}$, where $\phi_{2^j}(x, y) = (1/2^j)\phi(x/2^j, y/2^j)$. For a J-level, the collection $\{\mathcal{S}_{2^J}\tilde{f},\ \ \{\mathcal{W}^1_{2^j}\tilde{f},\ \mathcal{W}^2_{2^j}\tilde{f}\}\}_{1\le j\le J}$ is called the 2D discrete DWT. The gradient map of $\tilde{f}$ at pixel position (x, y) at scale 2^j is thus proportional to the wavelet transform modulus:

$$gradMap_{2^j}(\tilde{f})(x, y) = \sqrt{|\mathcal{W}^1_{2^j}\tilde{f}(x, y)|^2 + |\mathcal{W}^2_{2^j}\tilde{f}(x, y)|^2}, \tag{8}$$

and the angle of the gradient vector is given by $\mathcal{A}_{2^j}\tilde{f}(x, y) = \arctan(\mathcal{W}^2_{2^j}\tilde{f}(x, y)/\mathcal{W}^1_{2^j}\tilde{f}(x, y))$.