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A novel method for ranking of vague sets for handling the risk analysis of compressor system

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ABSTRACT

Vague sets were first proposed by Gau and Buehrer [11] as an extension of fuzzy sets which encompass fuzzy sets, inter-valued fuzzy sets, as special cases. Vague sets consist of two parts, that is, the membership function and nonmembership function. Therefore, in accordance with practical demand these sets are more flexible than the existing fuzzy sets and provide much more information about the situation. In this paper, a new approach for the ranking of trapezoidal vague sets is introduced. Shortcomings in some existing ranking approaches have been pointed out. Validation of the proposed ranking method has been established through the reasonable properties of the fuzzy quantities. Further, the proposed ranking approach is applied to develop a new method for dealing with vague risk analysis problems to find the probability of failure, of each component of compressor system, which could be used for managerial decision making and future system maintenance strategy. Also, the proposed method provides a useful way for handling vague risk analysis problems.

1. Introduction

Theory of fuzzy sets, proposed by Zadeh [39], has been successfully applied in various fields. Ranking between two fuzzy concept, as an important content in fuzzy mathematics, has gained attention for their wide applications in some areas such as decision making, transportation problems, risk analysis, aggregation and market prediction [4,10,12,15,19,26,29,30,33,35,38]. In past research the fuzzy numbers are usually used for evaluating the values of risk of each sub-component of the system. For example, Schmucker [28] proposed a method for fuzzy risk analysis based on fuzzy number arithmetic operations. Kangari and Riggs [14] proposed a method for constructing risk assessment using linguistic terms. Chen and Chen [5] proposed a method for fuzzy risk analysis based on similarity measures between generalized fuzzy numbers. Chen and Chen [6] presented a method for ranking generalized trapezoidal fuzzy numbers for handling fuzzy risk analysis problems. In recent years, the methods for ranking generalized fuzzy numbers have been extensively researched and used for dealing with fuzzy risk analysis problems. Ranking of fuzzy sets was first proposed by Jain [13]. Afterward, many authors developed several approaches for ranking fuzzy sets [27,29,31,37].

In the real world there are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to some reason evaluation of non-membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives; therefore, handling of these types of situations fuzzy sets theory [39] is not appropriate. A possible solution is to use intuitionistic fuzzy set [2] and vague set [11]. The ranking of vague sets plays a main role in real life problems involving vague decision-making, vague clustering, etc. Bustince and Burillo [3] pointed out that the notion of vague set is same as that of intuitionistic fuzzy set. Lu and Ng [22] proved that vague sets are more natural than intuitionistic fuzzy sets.

On the other hand, Mitchell [23] adopted a statistical viewpoint for the ranking of intuitionistic fuzzy number and interpret each intuitionistic fuzzy number as an ensemble of ordinary fuzzy numbers; this enables to define a fuzzy rank and a characteristic vagueness factor for each intuitionistic fuzzy number. Nayagam and Sivaraman [24] presented a new general accuracy function, which ranks all comparable interval valued intuitionistic fuzzy sets correctly. Also they suggested that the problem of ranking interval valued intuitionistic fuzzy sets is very much important in real life problems such as decision making, clustering and artificial

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intelligence, and their method is very much useful and has wide applications in all areas. Nehi [25] proposed a new ordering method for intuitionistic fuzzy numbers in which he consider two characteristic values of membership and non-membership for an intuitionistic fuzzy number. Li [20] proposed a new ranking method on the basis of the concept of a ratio of the value index to the ambiguity index and applied to multiattribute decision making problems in which the ratings of alternatives on attributes are expressed with triangular intuitionistic fuzzy numbers. Wei [34] introduced the ranking method for intuitionistic fuzzy numbers based on a possibility degree formula to compare two intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision making problems.

Due to the limitations in existing ranking method, it is necessary to develop the new ranking method for handling the risk analysis of compressor system. In this paper, a new approach for the ranking of trapezoidal vague sets is introduced. Validation of the proposed ranking method has been established through reasonable properties of the fuzzy quantities. Some shortcomings in existing ranking approaches have been pointed out. Also, the proposed ranking approach is applied to develop a new method for dealing with vague risk analysis problems to find the probability of failure, of each component of compressor system, which could be used for managerial decision making and future system maintenance strategy. The rest of the paper is organized as follows: Section 2 presents the introduction to vague sets, arithmetic operations between vague sets and reviews some existing ranking methods. In Section 3, some shortcomings of the existing ranking approaches have been pointed out. Section 4 introduces a new ranking approach for comparing vague sets. Section 5 validates the proposed ranking function through the reasonable properties for the ordering of fuzzy quantities and also, results of the proposed approach are compared with some existing approaches. Section 6 proposes a new approach for vague risk analysis of a compressor system based on the proposed ranking approach. Section 7 draws conclusions.

2. Preliminaries

In this section some basic definitions related to vague sets, arithmetic operations between vague sets and brief review of some existing ranking methods have been presented.

2.1. Vague sets

Definition 2.1. [11] A vague set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), 1 - \nu_{\tilde{A}}(x)) | x \in X\}$ on the universal set X is characterized by a truth membership function $\mu_{\tilde{A}}, \mu_{\tilde{A}} : X \to [0, 1]$ and a false membership function $\nu_{\tilde{A}}, \nu_{\tilde{A}} : X \to [0, 1]$. The values $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ represent the degree of membership and degree of non-membership of x and always satisfy the condition $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1 \forall \; x \in X$. The value $1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ represents the degree of hesitation of $x \in X$.

Definition 2.2. [11] A vague set $\tilde{A}$, defined on the universal set of real numbers R, denoted as $\tilde{A} = \langle[(a, b, c, d); \delta, \rho]\rangle$, where $a \leq b \leq c \leq d$ and $\delta \leq \rho$, is said to be a trapezoidal vague set if degree of membership, $\mu_{\tilde{A}}(x)$, and complement of the degree of non-membership, $(1 - \nu_{\tilde{A}}(x))$, are given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{\delta(x-b)}{(b-a)}, & a \leq x < b \\ \delta, & b \leq x \leq c \\ \dfrac{\delta(x-c)}{(d-c)}, & c < x < d \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad (1 - \nu_{\tilde{A}}(x)) = \begin{cases} \dfrac{\rho(x-b)}{(b-a)}, & a \leq x < b \\ \rho, & b \leq x \leq c \\ \dfrac{\rho(x-c)}{(d-c)}, & c < x < d \\ 0, & \text{otherwise} \end{cases}$$

where, $\delta = supremum\{\mu_{\tilde{A}}(x) : x \in R\}$ and $\rho = supremum\{(1 - \nu_{\tilde{A}}(x) : x \in R\}$.

Definition 2.3. A vague set $\tilde{A}$, defined on the universal set of real numbers R, denoted as $\tilde{A} = \langle[(a, b, c, d); \delta, \rho]_{LR}\rangle$, where $a \leq b \leq c \leq d$ and $\delta \leq \rho$, is said to be a LR type vague set if degree of membership, $\mu_{\tilde{A}}(x)$, and complement of the degree of non-membership, $(1 - \nu_{\tilde{A}}(x))$, are given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \delta L(\dfrac{b-x}{b-a}), & \text{for } a < x < b \\ \delta & \text{for } b \leq x \leq c \\ \delta R(\dfrac{x-c}{d-c}) & \text{for } c < x < d. \end{cases} \quad \text{and}$$

$$(1 - \nu_{\tilde{A}}(x)) = \begin{cases} \rho L(\dfrac{b-x}{b-a}), & \text{for } a < x < b \\ \rho & \text{for } b \leq x \leq c \\ \rho R(\dfrac{x-c}{d-c}) & \text{for } c < x < d. \end{cases}$$

The LR type vague set is a generalization of LR type fuzzy sets, where L and R stands for left membership function and right membership function respectively. LR type vague sets play a significant role in real life problems because it can deal with the uncertainty present within the membership functions. In other words, if the decision maker is not sure about the nature of available data, i.e., linear or non-linear, then LR type vague sets can be the solution.

2.2. Arithmetic operations

Let $\tilde{A} = \langle[(a_1, b_1, c_1, d_1); \delta_1, \rho_1]\rangle$ and $\tilde{B} = \langle[(a_2, b_2, c_2, d_2); \delta_2, \rho_2]\rangle$ be two trapezoidal vague sets, then

(i) $\tilde{A} \oplus \tilde{B} = \langle[(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); \min \; (\delta_1, \delta_2), \min(\rho_1, \rho_2)]\rangle$
(ii) $\tilde{A} \ominus \tilde{B} = \langle[(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); \min \; (\delta_1, \delta_2), \min(\rho_1, \rho_2)]\rangle$
(iii) $\gamma\tilde{A} = \begin{cases} \langle[(\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); \delta_1, \rho_1]\rangle, & \gamma \geq 0 \\ \langle[(\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); \delta_1, \rho_1]\rangle, & \gamma \leq 0 \end{cases}$
(iv) $\tilde{A} ø \tilde{B} = \langle[(a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); \min \; (\delta_1, \delta_2), \min(\rho_1, \rho_2)]\rangle$
In vague sets division, in order to avoid 0, we must limit $0 \notin [a_2, d_2]$.

2.3. Some existing ranking approaches

2.3.1. Li's ranking method [20]

Let $\tilde{A} = \langle(a_1, b_1, c_1); w_1, u_1\rangle$ and $\tilde{B} = \langle(a_2, b_2, c_2); w_2, u_2\rangle$ be two triangular intuitionistic fuzzy numbers, then use the following steps to compare $\tilde{A}$ and $\tilde{B}$:

Step 1 Let $V_\mu(\tilde{A})$ synthetically reflect the information on membership degrees, $V_\nu(\tilde{A})$ synthetically reflect the information on non-membership degrees. Also, $A_\mu(\tilde{A})$ and $A_\nu(\tilde{A})$ represent the ambiguities of the membership function $\mu(x)$ and the non-membership function $\nu(x)$, respectively for intuitionistic fuzzy