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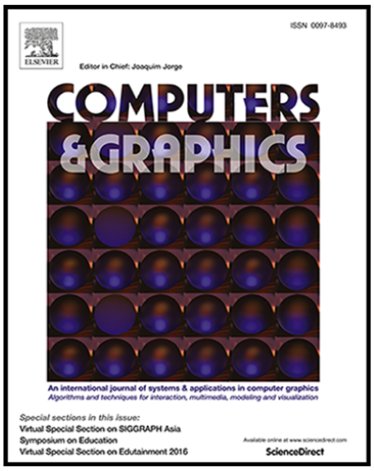

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On Visibility & Empty-Region Graphs

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Abstract

Empty-Region graphs are well-studied in Computer Graphics, Geometric Modeling, Computational Geometry, as well as in Robotics and Computer Vision. The vertices of these graphs are points in space, and two vertices are connected by an arc if there exists an empty region of a certain shape and size between them. In most of the graphs discussed in the literature, the empty region is assumed to be a circle or the union/intersection of circles. In this paper we propose a new type of empty-region graphs—the γ-visibility graph. This graph can accommodate a variety of shapes of empty regions and may be defined in any dimension. Interestingly, we will show that commonly-used shapes are a special case of our graph. In this sense, our graph generalizes some empty-region graphs. Though this paper is mostly theoretical, it may have practical implication—the numerous applications that make use of empty-region graphs would be able to select the best shape that suits the problem at hand.

1. Introduction

In this paper we re-visit empty-region graphs. These graphs are aimed at structural analysis of point sets [1]. Intuitively, a vertex of an empty-region graph represents a point in space and an arc connects two vertices if there exists an empty region of a certain shape and size between their respective points. These graphs have applications in computer vision [2, 3, 4], machine learning [5, 6], computer graphics [7], pattern classification [8], geographic analysis [13], as well as in networking [9] and in Bioinformatics [10].

We establish a novel link between two concepts in computer graphics: visibility of point clouds and empty-region graphs. This is done by defining a new empty-region graph, the γ-visibility graph, which connects the two. Differently from previous works, our graph accommodates a variety of shapes of empty regions. Therefore, though this paper is theoretical, it may find various uses in graphics and in robotics, as specific shapes of empty-regions may better suit specific problems. We further prove that our graph generalizes some commonly-used empty-region graphs.

We start by a short description of visibility of point sets. Given a point set, considered to be a sample of a continuous surface, and a viewpoint, the goal is to determine the sub-set of visible points. More precisely, since points cannot occlude each other, we are basically seeking a sub-set that would be visible to the viewpoint, if the surface from which the set of points was sampled, was known. The traditional way to perform the task is to reconstruct the surface from which the points are sampled and then determine visibility on the reconstructed surface.

However, in [11] an operator was introduced that determines visibility directly on the set, skipping reconstruction. The operator performs two steps: In Step 1, a function maps every point in the set to an inverted domain. In Step 2, the convex hull of the transformed points and the viewpoint is calculated. Points that reside on the convex hull of Step 2 turn out to be the pre-images of the visible points. In [12] the properties that should be satisfied by the function in Step 1 were identified and the operator was accordingly generalized to any function that satisfied these properties. This operator is termed the *Generalized Hidden Point Removal (GHPR) operator*.

We introduce in this paper a new graph structure, the *γ-visibility graph*. In this graph, two vertices are connected by an arc only if they are found to be visible to one another by the GHPR operator. This graph turns out to be an empty-region graph, as defined by [13]. Intuitively, this is so since indirectly, the GHPR operator "thresholds" the size of the empty regions between the viewpoint and the visible points. The shape of this region depends on the function applied in Step 1 and its size depends on the parameter γ of that function.

In contrast to most of the empty-region graphs proposed in the literature, in our case, the shape (template) region is not necessarily a union of circles or their intersection. Rather, it may take various shapes, which are determined by the function used in Step 1.

Our proposed γ-visibility graph has a couple of benefits. First, it generalizes empty-region graphs. The ability to define different shapes of empty regions makes it possible to match a specific shape to a specific application. For instance, a robot moving forward may not necessarily care about a circular empty region, but rather about a non-symmetric shape that emanates from the camera. Conversely, applications in communication may prefer circles. Second, we show that the Delaunay Triangulation is a special case of our γ-visibility graph.

The contributions of this paper are hence two-fold. First, we introduce a new and general graph structure, the γ-visibility graph and show how it provides a link between the class of visibility graphs to the class of empty-region graphs (Section 3). Second, we prove that the Delaunay triangulation is a special case of this graph (Section 4). This may have a couple of in-