Applied Soft Computing 26 (2015) 325–349

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Hesitant fuzzy Hamacher aggregation operators for multicriteria decision making

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ARTICLE INFO

Article history:
Received 22 July 2013
Received in revised form 6 October 2014
Accepted 8 October 2014
Available online 18 October 2014

Keywords:
Multicriteria decision making
Aggregation operator
Hesitant fuzzy set
Hesitant fuzzy element
Hamacher t-norm and t-conorm

ABSTRACT

As a fuzzy set extension, the hesitant set is effectively used to model situations where it is allowable to determine several possible membership degrees of an element to a set due to the ambiguity between different values. We first introduce some new operational rules of hesitant fuzzy sets based on the Hamacher t-norm and t-conorm, in which a family of hesitant fuzzy Hamacher operators is proposed for aggregating hesitant fuzzy information. Some basic properties of these proposed operators are given, and the relationships between them are shown in detail. We further discuss the interrelations between the proposed aggregation operators and the existing hesitant fuzzy aggregation operators. Applying the proposed hesitant fuzzy operators, we develop a new technique for hesitant fuzzy multicriteria decision making problems. Finally, the effectiveness of the proposed technique is illustrated by mean of a practical example.

1. Introduction

Various approaches to multiple criteria decision making require suitable aggregation functions [1–4]. This is particularly important when modeling different kinds of uncertainty using Zadeh's fuzzy set [5] and their higher order extensions, such as the interval-valued fuzzy set [6], intuitionistic fuzzy set [7,8], interval-valued intuitionistic fuzzy set [9], type-2 fuzzy set [10–12], type-n fuzzy set [10], and fuzzy multiset [12]. In some cases, it is difficult to determinate the degree of membership of an element to a set caused by ambiguity between a few different values. For example, a company may invite many experts to estimate the quality of an alternative with respect to a given criteria. Some experts regard 0.5 as the membership degree of quality, some regard 0.6 as the membership degree of quality, other regard 0.8 as the membership degree of quality. The three experts cannot compromise and change their evaluation. In this case, a simpler method is that the membership degree of quality should be composed of the three possible values. In order to deal with such situations, Torra and Narukawa [13] proposed the concept of the hesitant fuzzy set. For the above example, it can be represented by a hesitant fuzzy set $\{0.5, 0.6, 0.8\}$. As another generalized form of a fuzzy set, the hesitant fuzzy set permits several possible membership degrees of an element to a set, so it is an effective tool to represent a decision-maker's hesitancy in expressing his/her preferences.

After the pioneering work by Torra and Narukawa [13,14], the hesitant fuzzy set has been received more and more attention from researchers. Recently, it has been successfully applied to deal with uncertain multicriteria decision making problems. The hesitant fuzzy aggregation operator is one of the core issues. Based on some hesitant fuzzy operational rules, Xia and Xu [15] first proposed hesitant fuzzy weighted averaging (HFWA), hesitant fuzzy weighted geometric (HFWG) operators, generalized hesitant fuzzy weighted averaging (GHFWA), generalized hesitant fuzzy weighted geometric (GHFWG) operators, and applied them in solving decision making problems. Xu and Xia [16] proposed distance and similarity measures for hesitant fuzzy sets, and hesitant ordered weighted distance measures and hesitant ordered weighted similarity measures were investigated. The distance and correlation measures were further discussed for hesitant fuzzy information [17]. Extending quasi-arithmetic means to hesitant fuzzy sets, Xia et al. [18] proposed several new hesitant fuzzy aggregation operators for group decision making under hesitant fuzzy situations. Wei [19] extended the prioritized aggregation operator [20,21] to hesitant fuzzy sets, and developed some prioritized hesitant fuzzy operators for multicriteria decision making. According to the geometric Bonferroni mean, Zhu et al. [22] proposed a series of hesitant fuzzy geometric Bonferroni operators, such as the hesitant

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http://dx.doi.org/10.1016/j.asoc.2014.10.007
1568-4946/

fuzzy Choquet geometric Bonferroni mean, the weighted hesitant fuzzy geometric Bonferroni mean and the weighted hesitant fuzzy Choquet geometric Bonferroni mean operators, for multicriteria decision making problems with hesitant fuzzy information. Based on the hesitant fuzzy weighted averaging (HFWA) operator, Gu et al. [23] proposed a hesitant fuzzy multiple attribute decision making method to investigate risk investment problem. Wei et al. [24] extended the Chouquet integral to hesitant fuzzy sets, and proposed some hesitant fuzzy Choquet operators, such as the hesitant fuzzy Choquet ordered averaging and hesitant fuzzy Choquet ordered geometric operator, for interactive multicriteria decision making. A generalized hesitant fuzzy hybrid weighted distance measure was proposed by Wang et al. [25] to evaluate governmental archives website's construction. Zhang [26] developed a family of hesitant fuzzy power aggregation operators, for example the hesitant fuzzy power weighted average (HFPWA), hesitant fuzzy power weighted geometric (HFPWG), generalized hesitant fuzzy power weighted average (GHFPWA), generalized hesitant fuzzy power weighted geometric (GHFPWG) operators, and applied them to handle hesitant fuzzy multicriteria group decision making problems. A generalized hesitant fuzzy synergetic weighted distance measure was presented by Peng et al. [27] for multiple criteria decision making. Based on TOPSIS and maximizing the deviation method, Xu and Zhang [28] proposed a new approach for the hesitant fuzzy MCDM with incomplete weight information. Zhang and Wei [29] developed the E-VIKOR method and TOPSIS method to solve MCDM problems with hesitant fuzzy set information. Based on hesitant fuzzy linguistic term sets and context-free grammar, Rodriguez et al. [30,31] proposed a new linguistic group decision model that facilitated the elicitation of flexible and rich linguistic expressions. By means of the extending hesitant fuzzy set by the intuitionistic fuzzy set, Qian et al. [32] proposed a generalized hesitant fuzzy set, and applied it to decision support systems.

It is worthwhile to mention that these existing hesitant fuzzy aggregation operators, such as the HFWA, HFWG, GHFWA, GHFWG, HFPWA, HFPWG, GHFPWA, and GHFPWG operators, are based on the algebraic product and algebraic sum operational rules of hesitant fuzzy elements (HFEs), which are a pair of special dual triangular norm (briefly t-norm) and triangular conorm (briefly t-conorm) [3]. In the light of the relation between the intuitionistic fuzzy set and hesitant fuzzy set, general t-norm and t-conorm can be used to model the intersection and union of HFEs. That is to say, by means of t-conorm and t-norm, the above hesitant fuzzy aggregation operators are capable of being improved. For the intuitionistic fuzzy set, Beliakov et al. [33] constructed some operations about intuitionistic fuzzy sets based on Archimedean t-conorm and t-norm, and further extended generalized Bonferroni mean [34] to intuitionistic fuzzy sets, and proposed the generalized intuitionistic fuzzy Bonferroni operators [35]. Xia et al. [36] gave some operations of intuitionistic fuzzy sets based on Archimedean t-conorm and t-norm, and proposed an Archimedean t-conorm and t-norm based intuitionistic fuzzy weighted averaging operator, and an Archimedean t-cornorm and t-norm based the intuitionistic fuzzy geometric operator. Yu [37] proposed a generalized intuitionistic fuzzy prioritized weighted geometric operator based on Archimedean t-conorm and t-norm. Tan et al. [38] extended the quasi-OWA operator to the intuitionistic fuzzy set using Archimedean t-conorm and t-norm. However, these operators didn't consider the order weight and hybrid weight (i.e., integrating the attribute weight and the order weight), and they cannot be applied to HFEs, too. By means of another pair of special t-conorm and dual t-norm, the Einstein sum and Einstein product, Wang and Liu [39,40] proposed a wide range of intuitionistic fuzzy Einstein aggregation operators, such as the intuitionistic fuzzy Einstein weighted geometric (IFEWG) operator, the intuitionistic fuzzy Einstein ordered weighted geometric (IFEOWG) operator, the intuitionistic fuzzy Einstein weighted averaging (IFEWA) operator and the intuitionistic fuzzy Einstein ordered weighted averaging (IFEOWA) operator. According to the Einstein t-conorm and t-norm, Zhao and Wei [41] proposed some intuitionistic fuzzy hybrid Einstein aggregation operators for intuitionistic fuzzy multicriteria decision making. Yu [42] extended the Einstein t-conorm and t-norm to hesitant fuzzy sets, and proposed some hesitant fuzzy Einstein aggregation operators. Hamacher [43] proposed another kind of t-conorm and t-norm, called the Hamacher t-conorm and t-norm. As a generalization of the algebraic product and sum, and Einstein t-conorm and t-norm, the Hamacher t-conorm and t-norm is more general and more flexible [43]. In this paper, we extend the Hamacher t-conorm and t-norm to hesitant fuzzy set and propose a family of hesitant fuzzy Hamacher operators that allow decision-makers have more choice in multiple criteria decision making problems. It is also very important to research generalized hesitant fuzzy aggregation operators based on the Hamacher t-conorm and t-norm and their application to multicriteria decision making.

In order to do so, the paper will be set out as follows: In Section 2, some basic concepts of some aggregation operators, including t-norm and t-conorm, and hesitant fuzzy sets, are briefly reviewed. Some new hesitant fuzzy operational rules are introduced based on the Hamacher t-conorm and t-norm. In Section 3, based on these new Hamacher operational rules on hesitant fuzzy sets, we present a wide range of hesitant fuzzy Hamacher aggregation operators for hesitant fuzzy information, some of their desirable properties are investigated in detail, and the interrelations between the various operators are discussed. Further, we compare the proposed operators with the existing some hesitant fuzzy aggregation operators and obtain the corresponding relations. In Section 4, applying these proposed operators, we develop a technique for hesitant fuzzy multicriteria decision making. In Section 5, an example application of the new technique is provided, and we compare the developed method with the existing methods. In Section 6, we discuss the conclusions.

2. Preliminaries

2.1. Aggregation functions

Aggregation functions are used to combine a set of input values into a single representative output.

Definition 1 (*Aggregation operator* [2,34]). An aggregation function f: $[0,1]^n \to [0,1]$ is a function non-decreasing in each argument and satisfying $f(0, \ldots, 0) = 0$ and $f(1, \ldots, 1) = 1$.

The dual of an aggregation function can be expressed using the standard negation $N(t) = 1 - t$ and is denoted by $f_D(x_1, \ldots, x_n) = N(f(N(x_1), \ldots, N(x_n)))$.

Definition 2 (*GWA operator* [44]). A mapping f: $[0,1]^n \to [0,1]$ is called a generalized weighted averaging (GWA) operator of n-dimension if

$$f(x_1, x_2, \cdots, x_n) = \left(\sum_{i=1}^{n} w_i x_i^{\lambda} \right)^{1/\lambda},$$