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Computing urban radiation: A sparse matrix approach

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ABSTRACT

Cities numerical simulation including physical phenomena generates highly complex computational challenges. In this paper, we focus on the radiation exchange simulation on an urban scale, considering different types of cities. Observing that the matrix representing the view factors between buildings is sparse, we propose a new numerical model for radiation computation. This solution is based on the radiosity method. We show that the radiosity matrix associated with models composed of up to 140k patches can be stored in main memory, providing a promising avenue for further research. Moreover, a new technique is proposed for estimating the inverse of the radiosity matrix, accelerating the computation of radiation exchange. These techniques could help to consider the characteristics of the environment in building design, as well as assessing in the definition of city regulations related to urban construction.

1. Introduction

Due to the increasing need of energy assessment tools at large scale, urban physics simulation has become a major topic of interest. The evaluation of annual solar irradiance and the analysis of the spatial variation over building facades has a relevant interest for urban planning and building design. Computational simulation for radiative transfer on an urban scale is a challenge, because thousands of buildings have to be considered. The main problem is how to deal with the huge amount of data required to represent such models.

One of the mathematical models adapted to predict urban radiation exchange is the use of the radiosity method [1,2]. A full solution of this method in a city model may require computing the view factors between all building mesh elements and solving the linear system, which may be an expensive computational task when considering a district model composed of hundreds of buildings. A possible solution to manage the problem is to simplify the visibility problem [3].

We focus on solving the problem taking all visibility information into account. By observing that the form factor matrix that represents all view factors is sparse for this kind of environments, we propose a novel approach for radiative exchange computation that can approximate the inverse of the radiosity matrix. We formulate the problem as a Neumann series [4] and approximate the matrix by eliminating unimportant terms. Our study on different kinds of urban model configuration shows that, for models composed of thousands of patches, we can provide an accurate approximation of the inverse radiosity matrix that can also be stored in main memory. The radiosity method exposed here allows reducing the memory and execution time up to two orders of magnitude. This promising result enables processing city models bigger than 100k patches on a standard desktop PC. Moreover, the method can be applied for solving thousands of radiative configurations efficiently, considering many bounces of light and heat radiation. This is useful for light and heat calculations.

2. Related work

The two main methodologies for solving the urban radiant exchange problems are ray tracing and radiosity. While the former is widely used in rendering, the radiosity method is more suitable when the surfaces are Lambertian reflectors (such as concrete). One of the advantages of using this method is that it can give results in the whole scene space, which makes it attractive for urban environment analysis. In the rest of this section, we review the radiosity method and the works related to our approach.

2.1. The radiosity problem

The radiosity method [5] is a technique which allows computing global illumination on scenes with Lambertian surfaces. It has been applied in many areas of design and computer animation [6]. The continuous radiosity equation can be discretized through the

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use of a finite element methodology. The scene is discretized into a set of patches, leading to express the problem using the following set of linear equations:

$$B_i = E_i + R_i \sum_{j=1\dots n} B_j \mathbf{F}(i,j), \quad \forall i \in \{1\dots n\}$$

This set of linear equations is expressed in a succinct manner in Eq. (1).

$$(\mathbf{I} - \mathbf{RF})B = E, \tag{1}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{R}$ is a diagonal matrix containing the reflectivity index of each patch, B is the radiosity vector to be found, and E is the emission vector. $\mathbf{F}(i, j)$ is a number between 0 and 1 expressing the form factor between patch i and j. This value indicates the fraction of the light power going from one to another. Therefore, the form factor matrix is a $n \times n$ matrix, where n is the number of patches in the scene. $\mathbf{F}$ can be efficiently computed using the hemi-cube algorithm [7], but its memory requirements ($O(n^2)$) are often an obstacle when working with big models ($n >$ 50.000).

Eq. (1) can be solved using several approaches. For example, the operator $\mathbf{M} = (\mathbf{I} - \mathbf{RF})^{-1}$ can be calculated, which represents a global transport operator relating the emitted light with the final radiosity of the scene, $B = \mathbf{M}E$. When $\mathbf{F}$ has a low numerical rank, factorization techniques can be used to efficiently compute an approximation of $\mathbf{M}$ [8]. On the other hand, $\mathbf{M}$ can also be approximated using iterative methods such as Neumann series [4].

Another approach is to compute $\mathbf{B}$ by solving the linear system of equations iteratively, using methods such as Jacobi or Gauss–Seidel [1]. Eq. (2) presents the radiosity resolution using the Jacobi iteration. Each iteration adds the radiosity of a new light bounce to the global radiosity result.

$$B^{(i+1)} = \mathbf{RF}B^{(i)} + E, \quad \text{where } B^{(0)} = E \tag{2}$$

2.2. Correlation between scene characteristics and F properties

The characteristics of the analyzed scene model have a direct impact on the numerical properties of the associated $\mathbf{F}$ matrix. For example, in scenes with a high spatial coherence, matrices involved in radiosity calculations have a low numerical rank [8], because close patches have a high probability of being affected similarly by the rest of the scene. This fact enables the application of factorization techniques to compute low rank approximations that can be stored in main memory. On the other hand, in scenes with a high occlusion factor between patches, $\mathbf{F}$ can be efficiently represented using sparse representations. Two patches completely occluded do not exchange energy directly, and if this property is satisfied for most pair of patches on a scene, the form factors matrix has most of its elements equal to 0.

A *sparse matrix* is any matrix with enough zeros that it pays to take advantage of them [9]. Generally, using sparse representations allows saving time or memory (usually both) by exploiting the number of zeros. Furthermore, these kind of matrices are applied in problems where the use of full matrices is not possible due to memory limitations.

The use of sparse matrices in radiosity calculations is still a subject of study. Gortler et al. [10] present the Wavelet Radiosity method, which is based on wavelet theory. Expressing the kernel operating in a radiosity function in a wavelet basis leads to a sparse approximation of it. On the other side, Goel et al. [11], Borel et al. [12] and Chelle and Andrieu [13] solve the radiosity problem using iterative methods (like Gauss–Seidel) taking advantage of the sparsity of the form factors matrix. This property is present in the tested scenes (plant canopies), where there is a high occlusion level between distant polygons.

Studying the correlation between scene characteristics and $\mathbf{F}$ properties can help assessing the election of the correct technique for a given scene or sets of scenes. In this regard, there are scenes where neither sparse nor low-rank matrices are generated. Also, both sparse and low-rank $\mathbf{F}$ matrices could be associated with some kinds of scenes. Fig. 1 presents a diagram associated with these ideas, using four example models, each one with different properties. A picture of the scene, the sparsity structure of its associated $\mathbf{F}$, and a plot of its singular values are shown.

The upper left model of Fig. 1 corresponds to the plant canopy presented in [13]; the matrix $\mathbf{F}$ is sparse and its associated singular values decay is slow. The lower right scene is the Cornell box used in [8]; the matrix $\mathbf{F}$ is full and its numerical rank is low. The other two models were generated to test the existence of other scenes with different properties. The upper right scene is composed of several rooms, and connecting corridors. The rooms are simple boxes composed of a fine mesh, which makes them numerically low-rank by their own. Each room “sees” almost nothing of the others. This makes its form factors matrix sparse, as it can be seen in the figure, but its singular values decay fast enough to be considered numerically low-rank. On the other hand, the lower left scene represents an anechoic chamber, which is a room designed to absorb wave reflections. For this, its walls are filled with pyramids pointing inward. This particular property makes $\mathbf{F}$ to be highly dense and not numerically low-rank. The experimental results presented in Sections 3.1 and 4.2 suggest that city models have characteristics similar to plant canopy models.

2.3. Neumann series

Given an Operator $\mathbf{T}$, its Neumann series is a series of the form

$$\sum_{k=0}^{\infty} \mathbf{T}^k$$

The expression $\mathbf{T}^k$ is a mathematical notation that means applying the operator $\mathbf{T}$, k consecutive times. Supposing that $\mathbf{T}$ is a bounded operator and $\mathbf{I}$ the identity operator, if the Neumann series converges, then $(\mathbf{I} - \mathbf{T})$ is invertible and its inverse is the series:

$$(\mathbf{I} - \mathbf{T})^{-1} = \sum_{k=0}^{\infty} \mathbf{T}^k = \mathbf{I} + \mathbf{T} + \mathbf{T}^2 + \mathbf{T}^3 + \dots$$

This property can be used to calculate the radiosity [1], by computing an approximate to the inverse of $(\mathbf{I} - \mathbf{RF})$ through l iterations:

$$(\mathbf{I} - \mathbf{RF})^{-1} \approx \mathbf{I} + \mathbf{RF} + (\mathbf{RF})^2 + \dots + (\mathbf{RF})^l$$

In this series, $(\mathbf{RF})^i$ contains the information of the i^{th} bounce of light between the surfaces in the scene. The main computational cost of this approach is the multiplication of matrices. Thus, if $\mathbf{RF}$ is sufficiently big, the method could be too expensive.

[4] use a variant of this method to compute a global transport operator for radiance calculations. This operator expresses the relationship between the converged and incoming incident lighting. In this process, the matrices are compressed using the following strategy: at each step, all the coefficients below a certain threshold are removed. This results in sparse matrices, which allow speeding up the calculation. The computation is stopped when all the coefficients in $(\mathbf{RF})^i$ are smaller than the threshold.

2.4. Urban radiative methods

A previous work for reducing the urban radiosity formulation is the simplified radiosity algorithm (SRA) [3]. The basis of the simplification is grouping, for each sky direction, the main obstructions that obscured each surface. Then, for a scene composed of n