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ε-Guarantee of a covering of 2D domains using random-looking curves

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ABSTRACT

The problem of covering a given 2D convex domain D with a C^1 random-looking curve C is considered. C within D is said to cover D up to $\epsilon > 0$ if all points of D are within ϵ distance of C. This problem has applications, for example, in manufacturing, 3D printing, automated spray-painting, polishing, and also in devising a (pseudo) random patrol-path that will visit (i.e. cover) all of D using a sensor of ϵ distance span. Our distance bound approach enumerates the complete set of local distance extrema, enumeration that is used to provide a tight bound on the covering distance. This involves computing bi/tri-normals, or circles tangent to C at two/three different points, etc. A constructive algorithm is then proposed to iteratively refine and modify C until C covers a given convex domain D and examples are given to illustrate the effectiveness of our algorithm.

1. Introduction

Consider the problem of covering a given 2D domain by a random-looking parametric curve. Given a 2D domain D and a tolerance $\epsilon > 0$, the objective is to find a curve C within D such that each point in D is within ϵ distance of C. In other words, $\forall p \in D$, $\exists q \in C$, such that $\|p - q\| < \epsilon$. The need for covering of 2D domains by curves arises in many scenarios, viz., spray painting [2,16,21], automated polishing [5,14,19], tool path planning [7,9,18,22], rendering [10], surface inspection [17], and 3D printing [13]. In spray painting and polishing, the tool is required to paint/polish each point of the surface being worked upon. In tool path planning, the trajectory of the tool needs to be computed. The tool moves along this path, removing material from the block and yielding the desired workpiece. In surface inspection, the probe must traverse over the surface and inspect all the points. In all the above applications but [13], the covering curve is ordered, i.e., it either follows the isocurves of the surface being covered or is obtained as a solution of some variational optimization. In this work, we consider the problem of covering by random-looking curves. Our algorithm chooses the points affecting the shape of the curve on the fly, because the covering curve is constructed incrementally, adding new points in the regions that violate the ϵ-guarantee.

Recent advances in 3D printing technology have enabled manufacturing of a plethora of new kinds of artifacts. In particular, depositing material along a random curve leads to artistic objects [13]. A specific scenario, wherein random covering is useful, is in making artistic containers with minimal amount of material. For instance, say it is required to design a container for storing objects with minimal diameter greater than ϵ. This is achieved by having the surface of the container as the domain D and covering the same up to ϵ by a random curve C, which, when laid out by a 3D printer serves as the required container. Another application of random covering can be found in designing artistic patterns for fabrics and a photograph of such a carpet appears in Fig. 1. Yet another photograph of an artifact consisting of random curves is shown in Fig. 2, whose function is shown in Fig. 20.

Another interesting application of random covering is in designing paths for robot vacuum cleaners. A video from a proprietary source of such a robot in action can be seen in [1]. The creators of the robot claim that moving along a random path leads to better cleaning compared to moving along a systematic path, since many areas are visited more than once by the robot.

The problem of covering surfaces by curves is well studied. Elber et al. [9] proposed a formulation for covering surfaces by adaptively extracting isocurves, wherein, the total length of the curve is minimized. Tam [22] proposed an algorithm for uniform covering by curves along the isocurves of the surface. These are referred to as the scan-lines. The covering being uniform, a maximum distance is maintained between each pair of adjacent scan-lines. Antonio [2] address the problem of computing a trajectory for the spray applicator which minimizes the variation in the accumulated film thickness on the surface. This is done by formulating the problem as a constrained variational optimization problem. Rambhadran et al. [21] cast the problem of optimizing the time profile for spray painting as a constrained variational problem. The same is

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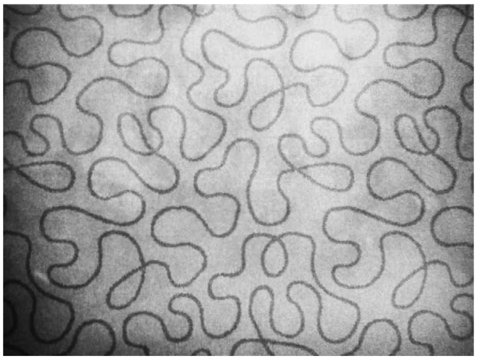

Fig. 1. A photograph of a carpet covered with random curves. A careful examination reveals that this pattern is not completely random.

Fig. 2. A photograph of an artifact covered using random curves.

then reduced to linear or quadratic programs. Cox et al. [7] use space-filling curves in the parameter domain of the surface to be machined. When mapped to the surface, this yields the tool path. Mizugaki [19] generate a planar Peano curve in the X-Y domain and map it to the surface of the workpiece via orthogonal projection, in order to generate the path for metal-mold polishing robot.

While there is a significant amount of work in covering surfaces with curves, to the best of our knowledge, only [13] examined the question of random covering. The main contributions of this work are twofold:

1. A method to evaluate the set of precise local extrema of distance from a curve within a 2D domain.
2. An algorithm which uses the above set of local distance extrema to iteratively construct a random-looking curve which covers the domain with ϵ-guarantee, for any given $\epsilon > 0$.

Our framework constructs C, in an iterative, random-looking manner. In each iteration, the set of local extrema of distance from C is computed and is used to refine C. In Section 2, we explain how these points of local extremum are computed. In Section 3, the overall algorithm is presented and shown to terminate. Several optimizations related to the running times and memory usage of our algorithm are proposed in Section 4. Results from the implementation of our algorithm in the IRIT [8] modeling environment are given in Section 5. We conclude the paper in Section 6, and also make remarks on possible extensions of this work.

2. Finding all distance local extrema from C

In this section, we formalize our distance bound approach by enumerating all the local extrema of the distance from C. We assume C is regular and C^1 continuous henceforth. The enumeration is done via two major cases. Section 2.1 analyzes the case when a local extremum point lies on the boundary, ∂D, of D. In Section 2.2, we explicate the case when the local extremum lies in the interior, D^o, of D. The points of local extrema thus computed must be subjected to a validity criteria, failing which, they are discarded. This is explained in Section 2.3.

In this work, D is assumed to be convex. Let closed interval I be the domain of the parametrization of C and t denote its parameter. The curve C is chosen to be open rather than closed since the ability to control the start and end position of C can add a useful degree of freedom, especially in highly complex and narrow containers and possibly in periodic tile covering. We will denote the Euclidean distance between two points $p, q \in D$ by $dist(p, q)$.

Definition 1. The **distance between a point** $p \in D$ **and** C is the distance between p and the point in C which is closest to p. We will denote this distance, again, by $dist(p, C)$. In other words,

$$dist(p, C) = \min_{t \in I} dist(p, C(t)).$$

Definition 2. Given a 2D domain D, a curve $C \subset D$ and $\epsilon > 0$, C **is said to cover** D **up to** ϵ if $\forall p \in D$, $dist(p, C) < \epsilon$.

Equivalently, $C \subset D$ is said to cover D up to ϵ if the farthest point of D from C is at distance less than ϵ. Such a point is a global extremum of the distance from C. Hence, if C covers D up to ϵ, then for each point $p \in D$, $\exists q \in C$ such that $dist(p, q) < \epsilon$.

Definition 3. Given a 2D domain D and a curve $C \subset D$, a point $p \in D$ is said to be a **local extrema of distance from** C if there exists a neighborhood $N \subset D$ of p such that either $d(p, C) \geq d(q, C)$, $\forall q \in N$ or $d(p, C) \leq d(q, C)$, $\forall q \in N$.

Our algorithm constructs the covering curve C iteratively and uses the (uniform) quadratic B-spline [6] representation for C as well as ∂D. ∂D is assumed to be piecewise C^1 continuous while C is C^1 continuous. Let C_i denote the curve at iteration i. In iteration i, a subset of the local extrema of the distance from C_i to points $p \in D$ affects the shape of C_{i+1}. The local extrema for $dist(p, C_i)$ are computed by solving a set of algebraic constraints as explained later in this section. If the farthest point from C_i is at a distance less than ϵ, the algorithm terminates. The farthest point in D from curve C is given by

$$p_0 = arg \max_{p \in D} dist(p, C),$$
$$= arg \max_{p \in D} \left\{ \min_{t \in I} dist(p, C(t)) \right\}.$$

The following lemma characterizes the set of local extrema of distance from C. Recall that D^o denotes the interior of D and ∂D the boundary of D. Then,

Lemma 4. *The set of local extrema of the distance from a C^1 curve C is completely enumerated as follows. If p_0 is a local extremum, then one of the following holds.*

1. *$p_0 \in D^o$ and p_0 is the center of a maximally inscribed circle with locally maximal radius which makes tangential/end-point contact with C at either two or three points.*
2. *$p_0 \in \partial D$ and p_0 is the center of a maximally inscribed circle with locally maximal radius which makes tangential/end-point contact with C at either one or two points.*

Proof. It is known that the local extrema of distance functions occur on the Medial Axis [3]. In our case, the set of local extrema of the distances from C occur on the Medial Axis of C. Further, the local extrema points of the Medial axis of some shape are centers of either bi-tangent or tri-tangent circles [20] to that shape. Hence, in our case, if a local extremum $p_0 \in D^o$, it is the center of a circle which makes contact with C at either two or three points and if p_0