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# Analysis of prismatic springs of non-circular coil shape and non-prismatic springs of circular coil shape by analytical and finite element methods ☆

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ABSTRACT

This paper presents a methodology for designing prismatic springs of non-circular coil shape and non-prismatic springs of circular coil shape using analytical and numerical methods. To start with, simple analytical formulations for obtaining the axial deformation of the springs under axial load have been demonstrated. Next, the processes of obtaining CAD models of the springs and their subsequent finite element analysis (FEA) in commercial softwares have been outlined. In the third part, the different springs have been compared with a common cylindrical spring and their merits compared to a common spring have been demonstrated. Next, a fairly accurate analytical formulation (with maximum error of ~7–8%) for obtaining the value and location of maximum shear stress for all the springs has been demonstrated. Next, two aspects of non-prismatic springs under dynamic loads, viz. damping introduced in a vibrating system and contribution of the spring to the equivalent mass in a one dimensional vibrating spring mass system due to shape of the spring have been discussed. The last part involves an analytical formulation for the linear elastic buckling of two springs with circular coil shapes. For the majority of the work, emphasis has been on obtaining and using closed form analytical expressions for different quantities while numerical techniques such as FEA have been used for validation of the same.



## 1. Introduction

The helical spring is one of the most fundamental flexible mechanical elements and mostly used in several industrial applications like balances, brakes, vehicle suspensions, and engine valves to satisfy functions like applying forces, storing or absorbing energy, providing the mechanical system with the flexibility and maintaining a force or a pressure. In addition, helical springs serve as the elastic member for most common types of vibration absorbers. The most commonly known helical spring, used in these applications, is presented as a cylindrical three-dimensional curved beam, characterized by its spiral shape and its constant curvatures along the axis. For these kinds of springs the demand of space in both lateral and vertical directions is undeniable. But for some very specialized applications, where there are lateral and (or) vertical space constraints, common springs may not be implemented with much success due to unwanted increase in stiffness mainly due to usage of multiple springs. This can be avoided by the usage of two special kinds of springs, viz. springs with non-circular shape to cater to restrictions in lateral space and springs of circular coil shape but non-prismatic profile to cater to restrictions in vertical space. Among the non-circular coil springs, the rectangular springs are used in light firearms. Among the non-prismatic springs, conical springs are generally used in applications requiring low solid height and increased resistance to surging, like automotive engines, large stamping presses, lawn mowers, medical devices, cell phones, electronics and sensitive instrumentation devices and volute shaped springs offer more lateral stability and less tendency to buckle than regular compression springs. Also, the possibility of resonance and excessive vibration (or surging) is reduced because volute springs have a uniform pitch, more damping due to coil structural (see Section 6.1) and an increasing natural period of vibration (instead of a constant period of vibration as in a cylindrical spring) as each coil closes.

For design and selection of springs for practical purposes, the deflection of the spring under axial load and maximum stresses induced are two major factors. Stress analysis is one of the main themes of research in helical springs. Investigations in this area began with the pioneering works of Ancker and Goodier (1958a, b), who used the boundary element method (not to be confused with the modern boundary element method) to apply theory of elasticity and to develop an approximate result to satisfy governing

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equations and boundary conditions along the surface of the coil. For small deformations of the spring, Wahl (1944) considered the wire of the spring as a round bar subjected to shear and torsion. The coupling between axial and torsional deformations was neglected in Wahl's approach and a correction factor was used to account for the curvature of the spring. Nagaya (1987) solved equations governing the distribution of stresses in the spring and developed an analytical approach but the aforesaid solution was applicable only for a few types of cross sections (circular, rectangular, etc.). Kamiya and Kita (1990) treated this problem also using boundary element method, and the analysis was limited to springs of small helix angle. Also, Cook (1990) analyzed the same type of springs by using finite element method and showed the limitation of the work associated with the methodology's negligence to helix angle of the spring. Haktanir (1994) solved the same problem by an analytical method to determinate the static stresses in the spring. Jiang and Henshall (2000) developed an approach based on the finite element method to analyze the stresses in a circular cross section helical spring by developing accurate boundary conditions and using finite element analysis. Fakhreddine, Mohamed, Said, Abderrazek, and Mohamed (2005) presented an efficient two-node finite element with six degrees of freedom per node, capable of modeling the total behavior of a helical spring.

In the approaches cited above, all the analyses were done considering only circular coil shaped prismatic springs of constant coil diameter. And the analyses and methods cited, albeit accurate, may not be easily used in cases where the spring coil is non-circular or the coil dimensions vary axially. But, as discussed before, springs of non-circular coil shape or non-prismatic springs find applications in practical cases when there is a limitation in space. Therefore, in the current work, analytical methods of obtaining the stress and deflection characteristics, two main design checkpoints for springs, have been attempted and the results obtained through the methodologies so developed have been compared with an independent method, FEA, to validate them.

The organization of the current work is as follows. Section 2 gives the analytical formulation for the deflection of prismatic and non prismatic springs under axial loads and benchmarks them against FEA. In Section 3, a brief discussion is presented on CAD representation of the springs in commercial softwares and FE analysis of the same using commercial softwares. In Section 4, the various springs discussed in Section 2 have been compared with a common prismatic spring with circular coils with an aim to point out the merits of the different springs. In Section 5, analytical expressions for obtaining the maximum stresses in the different springs have been presented and compared with FE analyses done using commercial softwares. The final section (Section 6) deals with the properties of the non-prismatic springs under dynamic loads and comparison of linear elastic buckling strengths of conical and right cylindrical springs of equivalent mass.

## 2. Deflection analysis of springs

In this section analytical methods for finding the deflections of different helical springs with constant pitch and wire diameter have been attempted. The formulation involves the usage of basic equations of solid mechanics, equilibrium of forces, and basic geometrical relationships. The results obtained from the formulations have been compared to those obtained from FEA of CAD models of the corresponding springs.

### 2.1. Deflection analysis of prismatic springs with non-circular coil shape

In this section, the analytical formulation for two varieties of prismatic springs with non-circular coil shape have been attempted. The prismatic springs have a uniform cross section through out length.

#### 2.1.1. Rectangular spring

In this section, a prismatic spring with a rectangular coil shape bounded by semicircles on the smaller sides (see Fig. 1a) has been attempted. The spring, although having an uncommon shape finds application in various mechanical equipments like guns and rifles. The basic dimensions of the profile of the spring is shown in Fig. 1. The length is $2a$ and the center of the circular arcs on either sides are coincident with the midpoint of the corresponding sides. The symbols as represented here will be followed throughout the section. It is seen that the profile is symmetric about each of the quadrants of axes on the plane with the origin coinciding with the geometric center of the figure. Advantage of this symmetry, shown in Fig. 1b, involving only the quarter of the coil shape is taken by deriving the relations for a quarter only and multiplying it by 4 for each of the coils. The straight part of the spring, shown in Fig. 1b, subtends an angle $\phi = \tan^{-1}\frac{a}{r}$ at the center of the coil. The force $F$, acting vertically at the center, induces both bending and torsional moment on a section of the coil. Expressions of moments in the circular and straight parts are different and are shown separately. On a section of the spring at a distance x from the vertical center line (see Fig. 1b), the bending and torsional moments, $M_x$ and $T_x$, induced by the force on the straight part are:

$$\left\{\begin{array}{l} M_x = Fr\tan(\theta) \\ T_x = Fr \end{array}\right\} \tag{1}$$

Also, from Fig. 2,

$$p = \sqrt{(a^2 + r^2 + 2ar\sin(\theta))} \tag{2}$$

$$\phi = \sin^{-1}\frac{r\cos(\theta)}{p} \tag{3}$$

Using above the values of bending and torsional moments, $M_\theta$ and $T_\theta$, induced by the force on the curved part are:

$$\left\{\begin{array}{l} M_\theta = Fp\sin(\theta+\phi) \\ T_\theta = Fp\cos(\theta+\phi) \end{array}\right\} \tag{4}$$

The total strain energy of the section shown in Fig. 2 is given by the sum of the strain energies due to the moments in the two separate sections $MN$ and $NQ$. Using Eqs. (1) and (4)

$$U_{sector} = \int_0^a \frac{M_x^2 dx}{2EI} + \int_0^a \frac{T_x^2 dx}{2GJ} + \int_0^{\frac{\pi}{2}} \frac{M_\theta^2 d\theta}{2EI} + \int_0^{\frac{\pi}{2}} \frac{T_\theta^2 d\theta}{2GJ} \tag{5}$$

where $I$ and $J$ represent the bending and torsional moments of inertia of the section of the wire with diameter $d$. $I = \frac{\pi d^4}{64}$, $J = \frac{\pi d^4}{32}$. $E$ and $G$ represent the Young's modulus and modulus of rigidity of the spring wire material. The total strain energy of the spring with $N_r$ number of active coils[1] may be given from Eq. (5) as $U_{Total} = 4N_r U_{sector}$, and the axial deflection of the spring due to the axial load $F$ as shown in Fig. 2, may be given as $\delta = \frac{\partial U_{Total}}{\partial F}$, following the well known Castigliano's theorem. A comparison of the above formulation and FEA of the same case is given below in Table 1. It has been assumed that $E = 210$ GPa for steel, the value of Poisson's ratio has been taken as $\nu = 0.25$ and wire diameter was taken as 3 mm. The spring under consideration has $N_r = 7.5$ for 8 complete turns with ground ends, and is under 15 N of axial load. From Table 1, it is seen that the analytical formulation for the deflection is in agreement with the FEA. Also, the closed form expression for the

[1] The number of active coils in a compression spring is generally less than the physical number of coils in the spring. It depends on the end conditions of the spring and a few other factors. For more details see the textbooks by Shigley (1972) or Bhandari (2010).