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Fuzzy exponentially weighted moving average control chart for univariate data with a real case application

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ABSTRACT

Statistical process control (SPC) is an approach to evaluate processes whether they are in statistical control or not. For this aim, control charts are generally used. Since sample data may include uncertainties coming from measurement systems and environmental conditions, fuzzy numbers and/or linguistic variables can be used to capture these uncertainties. In this paper, one of the most popular control charts, exponentially weighted moving average control chart (EWMA) for univariate data are developed under fuzzy environment. The fuzzy EWMA control charts (FEWMA) can be used for detecting small shifts in the data represented by fuzzy numbers. FEWMA decreases number of false decisions by providing flexibility on the control limits. The production process of plastic buttons is monitored with FEWMA in Turkey as a real application.

1. Introduction

A control chart is a tool that is commonly used to monitor and examine a process. It graphically depicts the average value and the upper and lower control limits of a process. The power of control charts lies in their ability to detect process shifts and to indicate abnormal conditions in a production process [11].

SPC is a powerful collection of problem solving tools for achieving process stability and improving capability through the reduction of variability. Traditional control charts are also known Shewhart control charts like individual, X-average and range, X-average and standard deviation for variable data. The exponentially weighted moving-average (EWMA) control chart is the best choice when we are interested in detecting small shifts.

The most common type of control charts used in a production process is the Shewhart control charts. Shewhart control charts, especially $\bar{X}$ and R charts may not be very sensitive for detecting shifts in the process mean or amount of process variation, particularly when the shift in the mean or in the variability is relatively small. Another weakness of $\bar{X}$ and R charts is that these two charts are not independent of each other. Also, there is a basic difference between the Shewhart control charts and the EWMA control charts in their method of construction and interpretation. The Shewhart charts plot independent sample data points, each of which is interpreted according to the probability law of the sampling distribution of the statistic in question. Inferences about possible shifts in the process parameters are made indirectly through the distribution patterns of the data on a Shewhart chart. An exponentially weighted moving statistic is directly an estimate of the corresponding process parameter. Therefore, a series of EWMA data on the chart tends to move slowly to the new level following a shift in the process, or will vary about the centerline with small fluctuations when the process is in control [2].

In the traditional control charts, data are composed of crisp values. But the measurement system that includes mainly operator and gage, and environmental conditions can be "uncertainty" or "vagueness" on crisp data. These uncertainties are based on the process and measurement system, which can lead to some difficulties in obtaining crisp values from the process. In this situation, fuzzy control charts are useful tools for evaluating fuzzy data. Fuzzy set theory can be adopted for the fuzzy control charts. In this condition, fuzzy set theory

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supports the development of concepts and techniques for dealing with sources of uncertainty or imprecision. Major contribution of fuzzy set theory is its capability to represent and modeling linguistic data and approximate data.

When human subjectivity plays an important role in defining the quality characteristics, the classical control charts may not be applicable since they require certain information. The major contribution of fuzzy set theory lies in its capability of representing vague data. Fuzzy logic offers a systematic base to deal with situations, which are ambiguous or not well defined. Fuzzy control charts are inevitable to use when the statistical data in consideration are uncertain or vague; or available information about the process is incomplete or includes human subjectivity [11].

Fuzzy control charts and its applications are well documented in literature. The fuzzy control charts were firstly presented by Raz and Wang [3] and Wang and Raz [4]. They proposed two approaches: probabilistic approach and membership approach. Kanagawa et al. [5] proposed control charts for linguistic terms as labeled fuzzy data from a stand point different to that of Wang and Raz [4] in order not only process average but also process variability. El-Shal and Morris [6] described an investigation into the use of fuzzy logic to modify SPC rules, with the aim of reducing the generation of false alarm and also improving the detection and detection-speed of real faults. Rowlands and Wang [7] introduced a method of fuzzy SPC evaluation and control which combines traditional statistical process control methodology with an intelligent system approach. Gülbay et al. [8] proposed α-cut control charts for attributes data to regulate the tightness of the inspection for attribute with triangular fuzzy numbers. Chen [9] proposed the method of constructing a fuzzy control chart for a process with fuzzy outcomes. Two fuzzy control charts have been constructed to directly monitor the fuzzy outcomes in order to establish whether or not the process is in control. Gülbay and Kahraman [10,11] proposed an alternative approach to fuzzy control chart: direct fuzzy approach. They used a direct fuzzy approach to fuzzy control chart for attributes under vague data using the probabilities of fuzzy events. Faraz and Moghadam [12] introduced a fuzzy chart for controlling the process mean. They designed the fuzzy chart that has a warning line besides upper control limit. Erginel [13] showed the theoretical structure of fuzzy individual and moving range control charts with α-cuts by using α-level fuzzy median transformation techniques. Şentürk and Erginel [14] firstly presented the theoretical structure of fuzzy $\tilde{\bar{X}}-\tilde{R}$ and $\tilde{\bar{X}}-\tilde{S}$ control charts with α-cuts by using α-level fuzzy midrange transformation techniques. They used the triangular fuzzy membership functions with fuzzy numbers (a, b, c) to obtain the fuzzy $\tilde{\bar{X}}-\tilde{R}$ and $\tilde{\bar{X}}-\tilde{S}$ control charts. Şentürk [15] also presented fuzzy regression control chart and Şentürk et al. [16] developed fuzzy $\tilde{u}$ control charts. Kaya and Kahraman [25] derived fuzzy control charts for fuzzy measurements of the related quality characteristics. The fuzzy control charts were used to increase the accuracy of process capability analysis by determining whether or not the process is in statistical control.

Fuzzy multivariate exponentially weighted moving average (F-MEWMA) control chart proposed by Alipour and Noorossana [18] and applied to food industry.

In this paper, a fuzzy exponentially weighted moving average control chart (FEWMA) is developed under fuzzy environment for univariate data. The main contribution of this study is to develop the theoretical base of the fuzzy exponentially weighted moving average control charts (FEWMAs) with α-cuts for univariate data. When the data are linguistic and detecting the small shifts are required, the fuzzy EWMA control chart is indispensable statistical process control tool. For modeling fuzzy EWMA control charts, triangular fuzzy numbers are used and fuzzy data are transformed by α-level fuzzy median transformation technique.

The rest of the paper is organized in the following order. The definitions of fuzzy numbers and fuzzy transformation techniques in literature are presented in Section 2. The fuzzy EWMA control charts with α-cuts are given in Section 3. A real case application on a plastic button in clothing industry is given in Section 4. Conclusions are given in Section 5.

2. Fuzzy numbers and fuzzy transformation techniques

Uncertainty and vagueness arises from ignorance, from chance, from various classes of randomness, from imprecision, from the inability to perform adequate measurements, from lack of knowledge, like the fuzziness inherent in our natural language. Fuzzy sets provide a mathematical way to represent vagueness in humanistic systems [20]. α-cuts approximation is used when obtaining the formulation of fuzzy EWMA control chart. In our fuzzy set approach, the samples taken from a process are represented by triangular membership functions as given in Fig. 1.

When fuzzy data were used, it is necessary to represent the fuzzy sets associated with these linguistic data in the sample by some representative (transformation) numbers for further calculations. The four fuzzy measures of central tendency used in descriptive statistics are given as following [4]:

The fuzzy mode, f_{mode}: The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals 1. This is stated as:

$$f_{\text{mode}} = \{x|\mu_F(x) = 1\}, \quad \forall x \in F. \tag{1}$$

It is unique if the membership function is unimodal.

The α-level fuzzy midrange, f_{mr}^{α}: The average of the end points of an α-cut. An α-cut, denoted by F_α, is a non-fuzzy subset of the base variable x containing all the values with membership function values greater than or equal to α. Thus $F_\alpha = \{x|\mu_F(x) \geq \alpha\}$. If a_α and c_α are end points of α-cut F_α such that $a_\alpha = Min\,\{F_\alpha\}$ and $c_\alpha = Max\,\{F_\alpha\}$, then,

$$f_{mr}^{\alpha} = \frac{1}{2}(a_\alpha + c_\alpha) \tag{2}$$

The fuzzy median, f_{med}: This is the point that partitions the curve under the membership function of a fuzzy set into two equal regions satisfying the following equation:

$$\int_a^{f_{\text{med}}} \mu_F(x)dx = \int_{f_{\text{med}}}^c \mu_F(x)dx = \frac{1}{2}\int_a^c \mu_F(x)dx \tag{3}$$

where a and c are the end points in the base variable of the fuzzy set F such that $a < c$.