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Aggregating information and ranking alternatives in decision making with intuitionistic multiplicative preference relations

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ABSTRACT

The intuitionistic multiplicative preference relation (IMPR), whose all elements are measured by an unsymmetrical scale (Saaty's 1–9 scale) instead of the symmetrical scale in the intuitionistic fuzzy preference relation (IFPR), is suitable for describing the asymmetric preference information. In decision making process, one of the most crucial issues is how to rank alternatives from the given preference relation constructed by the decision maker. In this paper, two approaches are proposed for deriving the ranking orders of the alternatives from two different angles. To do it, a transformation mechanism is developed to transform an IMPR to a corresponding IFPR, and then all alternatives depicted by the given IMPR can be ranked via solving a familiar IFPR. In addition, the generalized intuitionistic multiplicative ordered weighted averaging (GIMOWA) and the geometric (GIMOWG) operators are given by taking fully account of the different weights associated with the particular ordered positions and their desirable properties are also discussed. After that, through a practical example, the proposed approaches are compared with the previous work and a numerical analysis of the results is also given.

1. Introduction

In a decision making problem, a finite set of alternatives is usually to be evaluated and ranked, and perhaps at least one decision maker (DM) is invited to provide his/her preferences over the alternatives. A common way to settle the problem contains two steps: The first step called "construction step" is to construct a preference relation which is the most usual case to represent the DM's preferences, and the second one called "exploitation step" is to rank the alternatives from the constructed preference relation. In the "construction step", the DM compares the considered alternatives by pairs, and constructs a preference relation. There are two of the most widely used preference relations: the fuzzy preference relation [1] and multiplicative preference relation [2]. A fuzzy preference relation is defined as a complementary matrix which assumes that the grades between "Extremely not preferred" and "Extremely preferred" are distributed uniformly and symmetrically; while a multiplicative preference relation is defined as a reciprocal matrix which is not uniform and symmetrical [3–7]. In addition, all elements in a fuzzy and multiplicative preference relation are only characterized by a membership function describing the strength that one alternative is preferred to another, and cannot consider the degree that one alternative is not preferred to another. To solve such cases, the intuitionistic fuzzy preference relation (IFPR) [8] and the intuitionistic multiplicative preference relation (IMPR) [9] are defined to simultaneously depict the degree that one alternative is preferred to another and the degree that one alternative is not preferred to another. Especially, the IMPR recently proposed by Xia et al. [9] can deal with the unbalanced distribution commonly found in real-life. One example is the law of diminishing marginal utility in economics [9–11]. When increasing the same amount of investment, a company with worse performance yields more utility than that with better performance. Another example is the rain attenuation prediction for satellite communication. According to the prediction model in ITU-R (International Telecommunication Union-Radio Communication Sector) [12], with improving a certain frequency, it increases more rain attenuation in the higher operating frequencies than in the lower frequencies, and causes more cost to compensate the signal outage. Generally speaking, sometimes the gap between the grades expressing good information should be larger than the one between the grades reflecting bad information. Due to the advantages of IMPRs, it is necessary and interesting to

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Table 1
The Saaty's 1–9 scale.

Saaty's 1–9 scale	Meaning
1/9	Extremely not preferred
1/7	Very strongly not preferred
1/5	Strongly not preferred
1/3	Moderately not preferred
1	Equally preferred
3	Moderately preferred
5	Strongly preferred
7	Very strongly preferred
9	Extremely preferred
Other values between 1/9 and 9	Intermediate values used to present compromise

investigate the approaches for ranking alternatives from IMPRs in decision making. Up to now, some works have been achieved: Xia et al. [9] introduced the concept of IMPR and developed the aggregation principle of the intuitionistic multiplicative preference information for decision making; Xu [13] developed a method to derive the priority weights of the objects from an IMPR; Jiang et al. [10] defined the compatibility measure for IMPRs and developed two consensus models for group decision making; Xia and Xu [11] introduced some intuitionistic multiplicative aggregation operators and applied them into group decision making with IMPRs. Based on algebraic operational laws, Yu and Fang [14] developed some intuitionistic multiplicative aggregation operators and introduced a method for group decision making with IMPRs.

It is noticed that, especially in the "exploitation step" with IMPRs, most of the above work focuses on the aggregation operators, and most of these aggregation operators take little account of the different weights of the particular ordered positions of alternatives (except the intuitionistic multiplicative order weighted averaging operator based on algebraic operational laws (A-IMOWA operator) proposed by Yu and Fang [14]). Different from IMPRs, series of papers [11,15–21] have developed various exploitation techniques (rather than aggregation operators) for ranking alternatives based on IFPRs. Hence, one aim of this paper is to explore the relationship between IMPRs and IFPRs. Then the existing work on the IFPRs can be applied into dealing with the IMPRs by taking use of such relationship. Another aim of this paper is to introduce some "ordered" weighted operators which are helpful for relieving the influence of unfair information by assigning low weights to those "biased" arguments, like the generalized intuitionistic multiplicative ordered weighted averaging (GIMOWA) and the geometric (GIMOWG) operators, by extending the results in Xia et al. [9]. To do this, the rest of the paper is arranged as follows: Section 2 reviews some basic knowledge. In Section 3, a transformation mechanism between IMPRs and IFPRs is proposed, based on which, an approach for ranking alternatives in decision making with IMPRs is presented. In Section 4, some new ordered weighted aggregation operators for aggregating intuitionistic multiplicative information are developed and applied to derive the ranking of the given alternatives. Section 5 provides a practical example to illustrate the proposed methods and compare them with the previous work. Concluding remarks and further research directions are included in Section 6.

2. Basic concepts

To facilitate the presentation, firstly, some basic definitions and concepts are reviewed. For simplicity, this paper denotes $N = \{1, 2, \ldots, n\}$, $M = \{1, 2, \ldots, m\}$ and $X = \{x_1, x_2, \ldots x_n\}$.

Definition 2.1. [1]. A fuzzy preference relation R on the set X is defined as a reciprocal complementary matrix $R = (r_{ij})_{n\times n} \subset X \times X$, which satisfies

$$0 \le r_{ij} \le 1, \quad r_{ij} + r_{ji} = 1, \quad r_{ii} = 0.5, \quad \text{for all } i, j \in N$$

where r_{ij} denotes the preference degree of the alternative x_i over x_j.

Different from the fuzzy preference relation which uses the 0–1 scale, the multiplicative preference relation is measured by the 1/9–9 scale (also called Saaty's 1–9 scale) as follows:

Definition 2.2. [2]. A multiplicative preference relation P on the set X is defined as a reciprocal matrix $P = (p_{ij})_{n\times n} \subset X \times X$, which satisfies

$$1/9 \le p_{ij} \le 9, \quad p_{ij} \cdot p_{ji} = 1, \quad p_{ii} = 1, \quad \text{for all } i, j \in N$$

where p_{ij} is interpreted as the ratio of the preference intensity of the alternative x_i to that of x_j.

This paper focuses on the multiplicative preference relations in which all information is measured by the 1/9–9 scale, whose meanings are listed in Table 1.

Form Table 1, it is easily found that $p_{ij} = 1$ indicates indifference between x_i and x_j; $p_{ij} > 1$ indicates the degree that x_i is preferred to x_j, the greater p_{ij}, the stronger the preference intensity of x_i over x_j, especially $p_{ij} = 9$ indicates that x_i is extremely preferred to x_j; $p_{ij} < 1$ means that the degree that x_j is preferred to x_i, the smaller p_{ij}, the stronger the preference intensity of x_j over x_i, especially $p_{ij} = 1/9$ means that x_j is extremely preferred to x_i.

By considering the non-membership information in the fuzzy preference relation, the IFPR is defined as follows:

Definition 2.3. [8]. An intuitionistic fuzzy preference relation (IFPR) B on the set X is represented by a matrix $B = (b_{ij})_{n\times n}$, where $b_{ij} = (\mu_{b_{ij}}, \nu_{b_{ij}})$ is an intuitionistic fuzzy number (IFN) for all $i, j \in N$, and $\mu_{b_{ij}}$ is the certainty degree to which x_i is preferred to x_j, $\nu_{b_{ij}}$ is the certainty degree to which x_i is not preferred to x_j and both of them satisfy

$$\mu_{ji} = \nu_{ij}, \quad \nu_{ji} = \mu_{ij}, \quad \mu_{ii} = \nu_{ii} = 0.5, \quad 0 \le \mu_{ij} + \nu_{ij} \le 1, \quad 0 \le \mu_{ij}, \nu_{ij} \le 1$$