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# A risk attitudinal ranking method for interval-valued intuitionistic fuzzy numbers based on novel attitudinal expected score and accuracy functions

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ABSTRACT

This article investigates new score and accuracy functions for ranking interval-valued intuitionistic fuzzy numbers (IVIFNs). The novelty of these functions is that they allow the comparison of IVIFNs by taking into account of the decision makers' attitudinal character. The new attitudinal expected score and accuracy functions extend Xu and Chen's score and accuracy degree functions, and verify the following set of properties: (1) boundedness; (2) monotonicity; (3) commutativity; and (4) symmetry. These novel functions are used to propose a total order on the set of IVIFNs, and to develop an interval-valued intuitionistic fuzzy multi-attribute decision making selection process in which the final result depends on the decision maker's risk attitude. In addition, a ranking sensitivity analysis with respect to the risk attitude is provided.



## Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy sets (IFSs), which is characterised by both the membership and non-membership functions and therefore it generalises the concept of fuzzy set [2,3]. Subsequently, Atanassov and Gargov [4] extended IFSs with the introduction of the concept of interval-valued intuitionistic fuzzy sets (IVIFSs). The notions of IFSs and IVIFSs are interesting and very useful in modelling real life problems with imprecision or uncertainty and they have been applied to many different fields, including multiple attribute decision making (MADM) [5–9], group decision making (GDM) [10–12], supplier selection [13,14], robot selection [15] and artificial intelligence [16].

The first step of any MADM process with information modelled using IFSs is to fuse the intuitionistic fuzzy assessment values of the different attributes into a collective intuitionistic fuzzy assessment via an appropriate aggregation operator [17]. Once this step has been completed, the aggregated intuitionistic fuzzy numbers are compared to produce a final ranking of the alternatives. Consequently, an active research topic is the investigation of intuitionistic fuzzy MADM that includes suitable and valid intuitionistic fuzzy aggregation operators. Since Xu [18] developed the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, extensive research work has been carried out to develop aggregation operators for both IFSs and/or IVIFSs (see for example [19–21]). However, the above cited intuitionistic fuzzy operators are based on additive measures and are not suitable to aggregate inter-dependent criteria. To resolve this issue, Tan and Chen [22,23] proposed the intuitionistic fuzzy Choquet integral operator and the generalised interval-valued intuitionistic fuzzy geometric aggregation operator for multi-attribute interval-valued intuitionistic fuzzy group decision making problems.

Another active research topic regards the development of score degree and accuracy degree functions to make possible the comparison of criterion values that are expressed by IFSs and IVIFSs, respectively. A comprehensive comparative analysis of existing score degree and accuracy degree functions to date can be found in [24]. Chen and Tan in [25] developed a score degree function for IFSs based on the membership degree and non-membership degree functions, which was later improved by Hong and Choi in [26] with the addition of an accuracy degree function. In addition, Hong and Choi argued about the similarity between the role of the score degree and the accuracy

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degree functions of IFSs and that of the mean and the variance in statistics. Subsequently, other improved score degree and accuracy degree functions have been proposed in [27–32]. Also these functions are extended to the cases of triangular intuitionistic fuzzy numbers [33,34] and intuitionistic linguistic numbers [35–37]. It is worth mentioning the score degree and accuracy degree functions recently developed by Xu and Chen in [38] to propose an order relation on the set of interval-valued intuitionistic fuzzy numbers (IVIFNs). However, as it will be proved later with a counter-example (Example 1), the order relation derived from the application of Xu and Chen's score degree and accuracy degree functions is not total. In a attempt to resolve this drawback, Ye in [39] and Lakshmana Gomathi Nayagam and Sivaraman in [40] proposed alternative accuracy degree functions, respectively, which they claimed produced a total ordering of IVIFNs. However, these alternative degree functions are not superior to the existing accuracy degree functions but equivalent 'the course of comparing any two interval-valued intuitionistic fuzzy numbers'. As it will be shown later in section 'Ordering relation of interval-valued intuitionistic fuzzy numbers', these score degree and accuracy degree functions do not capture well all the information contained in IVIFNs and consequently can lead to a lack of precision in the final ordering of IVIFNs. We believe that this is because these functions are simple and straight forward extensions of their respective proposals for the case of IFNs. An important limitation of the above approaches resides in the fact that they do not take into account the attitudinal character of decision makers. Yager in [41] pointed out that the attitudinal character of each decision maker may affect the final ranking order of fuzzy numbers (FNs). The problem of ordering FNs, though, has been extensively studied and an agreed conclusion is that there is no unique best approach to do this. Recall that FNs are particular cases of IVIFNs. Thus, the same conclusion applies to IVIFNs. Therefore, it is important to develop a methodology that best captures the decision maker's risk attitude regarding the ranking IVIFNs.

In order to achieve this, the the remainder of this paper is organised as follows: The next section briefly reviews the main score degree and accuracy degree functions of IVIFSs and an analysis of their relationships as well as their associated drawbacks is carried out, which it is used in section 'The risk attitudinal expected score and accuracy functions' to support the development of novel attitudinal expected score and accuracy functions of IVIFNs driven by the decision maker's attitudinal character. In this section, it is also proved that the new attitudinal expected functions extend Xu and Chen's score degree and accuracy degree functions. The following desirable properties are proved to be satisfied by the new attitudinal expected functions: (i) boundedness; (ii) monotonicity; (iii) commutativity; and (iv) symmetry. This section is completed with the definition of a total order relation on the set of IVIFNs. Section 'Interval-valued intuitionistic fuzzy multi-attribute decision-making method based on the attitudinal expected functions' presents a sensitivity analysis with respect to the attitudinal character and a resolution process of MADM problems in an interval-valued intuitionistic fuzzy environment. Finally, in section 'Conclusion' conclusions are drawn and suggestions made for further work.

## Preliminaries

This section presents the key concepts related to IVIFSs that will be used throughout this paper. First, we present Atanassov and Gargov's definition of the notion of IVIFS, which is characterised by a membership function and a non-membership function that take interval numbers rather than crisp numbers, as introduced in [4].

**Definition 1** *(Interval-valued IFS (IVIFS)).* Let $INT([0, 1])$ be the set of all closed subintervals of the unit interval and $X$ be a universe of discourse. An interval-valued IFS (IVIFS) $A$ over $X$ is given as:

$$A = \left\{ \langle x, \widetilde{\mu}_A(x), \widetilde{\nu}_A(x)\rangle | x \in X \right\} \tag{1}$$

where $\widetilde{\mu}_A(x), \widetilde{\nu}_A(x) \in INT([0, 1])$, represent the membership and the non-membership degrees of the element $x$ to the set $A$ subject to the following constraint

$$0 \le \sup \widetilde{\mu}_A(x) + \sup \widetilde{\nu}_A(x) \le 1, \quad \forall x \in X.$$

Denoting by $\widetilde{\mu}_{AL}(x), \widetilde{\mu}_{AU}(x), \widetilde{\nu}_{AL}(x)$ and $\widetilde{\nu}_{AU}(x)$ the lower and upper end points of $\widetilde{\mu}_A(x)$ and $\widetilde{\nu}_A(x)$, respectively, an IVIFS can be represented as

$$A = \left\{ \langle x, [\widetilde{\mu}_{AL}(x), \widetilde{\mu}_{AU}(x)], [\widetilde{\nu}_{AL}(x), \widetilde{\nu}_{AU}(x)]\rangle | x \in X : 0 \le \widetilde{\mu}_{AU}(x) + \widetilde{\nu}_{AU}(x) \le 1, \widetilde{\mu}_{AL}(x) \wedge \widetilde{\nu}_{AL}(x) \ge 0 \right\} \tag{2}$$

Recall that given two IVIFSs, $A$ and $B$, Atanassov and Gargov containment concept is modelled as follows [4]:

$$A \subseteq B \quad \text{iff} \quad \forall x \in X : \widetilde{\mu}_{AL}(x) \le \widetilde{\mu}_{BL}(x) \quad \wedge \quad \widetilde{\mu}_{AU}(x) \le \widetilde{\mu}_{BU}(x) \quad \wedge \quad \widetilde{\nu}_{AL}(x) \ge \widetilde{\nu}_{BL}(x) \quad \wedge \quad \widetilde{\nu}_{AU}(x) \ge \widetilde{\nu}_{BU}(x).$$

The hesitancy degree function of an IVIFS is:

$$\widetilde{\pi}_A(x) = [1 - \widetilde{\mu}_{AU}(x) - \widetilde{\nu}_{AU}(x), 1 - \widetilde{\mu}_{AL}(x) - \widetilde{\nu}_{AL}(x)]. \tag{3}$$

Given $x \in X$,

$$([\widetilde{\mu}_{AL}(x), \widetilde{\mu}_{AU}(x)], [\widetilde{\nu}_{AL}(x), \widetilde{\nu}_{AU}(x)])$$

will be referred to as an interval-valued intuitionistic fuzzy number (IVIFN). For convenience, an IVIFN will be denoted by $([\mu^-, \mu^+], [\nu^-, \nu^+])$.

Given two IVIFNs $\widetilde{\alpha}_1 = ([\mu_1^-, \mu_1^+], [\nu_1^-, \nu_1^+])$ and $\widetilde{\alpha}_2 = ([\mu_2^-, \mu_2^+], [\nu_2^-, \nu_2^+])$, we have the following definition of containment [4]:

$$\widetilde{\alpha}_1 \subseteq \widetilde{\alpha}_2 \quad \text{iff} \quad \mu_1^- \le \mu_2^-, \quad \mu_1^+ \le \mu_2^+, \nu_1^- \ge \nu_2^-, \text{ and } \nu_1^+ \ge \nu_2^+.$$

Also, the main arithmetic operations can be expressed in terms of the interval lower and upper bounds as follows [1,42]: