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Network-coded cooperative communications with multiple relay nodes: Achievable rate and network optimization

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ABSTRACT

Network-coded cooperative communications (NC-CC) refers to the use of network coding (NC) in cooperative communications (CC). Prior studies have shown that NC has the potential to improve the performance of CC when there are multiple sessions in the wireless network. These studies were done for the case when multiple sessions are sharing a single relay node. However, how NC-CC behaves when multiple relay nodes are employed remains an open problem. In this paper, we explore this problem by analyzing the achievable rate of each session in this setting. We develop closed form formulas for the mutual information and the achievable data rate for each session. We show that prior results for a single relay is a special case of our result. Based on these findings, we then study a network optimization problem that requires joint optimization of session grouping, relay node grouping, and matching of session/relay groups. We show that this problem is NP-hard, and present a polynomial time heuristic algorithm to solve this problem. Using simulation results, we show this algorithm is highly competitive and can produce results that are near to optimality.

1. Introduction

Cooperative communications (CC) is an important technique to improve the performance of a wireless network [11]. Unlike MIMO, which requires the use of multiple antennas at each node, CC only employs a single antenna at each node and exploits diversity by cooperating with antennas on other nodes in the network. CC schemes can be amplify-and-forward (AF) or decode-and-forward (DF) [15]. Under AF, the relay simply amplifies its received signal while under DF, the relay decodes its received signal and then encodes data again before forwarding to the destination. There has been extensive research at the physical layer that exploits cooperations among distributed antennas [1,6,13].

Recently, it was found that network coding (NC) can further improve the performance of CC by combining data streams at a relay node [4,18–25,29,33–36]. This application of NC in CC is called network-coded CC (or NC-CC). NC has been shown to improve the outage probability [4,19], packet error rates [33], and data rates [25,36] for CC. NC schemes can be either digital or analog [10], depends on whether network coding is done on digital signals or analog signals. Most of these studies were done for the case when multiple sessions are sharing a single relay node [4,19,25,33,36] or multiple sessions have the same destination [18,20–24,34,35]. Topakkaya and Wang [29] considered the scenario of multiple source-destination pairs using multiple relays, designed a network coding scheme, and analyzed its performance. In this paper, we will design another network coding scheme with less number of time slots in a frame and thus may achieve larger rate.

In this paper, we study NC-CC when there are multiple relay nodes. Our goal is two-fold.

First, we aim to develop closed form formulas for the mutual information and the achievable data rate for each session. We consider the case of analog network coding (ANC) [10] and AF CC [15] at each relay node. Through an in-depth analysis, we derive the mathematical equations for mutual information and achievable data rate for each session and show that prior results for a single relay is a special case of our result. This finding offers an important building block on the theory of NC-CC.

Second, we investigate the following important problems jointly in a multi-user network under NC-CC: (i) how to put sessions into different groups; (ii) how to put relay nodes into different groups; and (iii) how to match the session groups with relay groups under NC-CC. Specifically, we study a network optimization problem with the goal of maximizing the sum of weighted rates of all sessions. This optimization problem requires a joint optimization of all three

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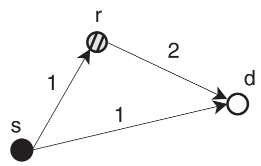


Fig. 1. A three-node relay channel for CC.

Table 1
Notation.

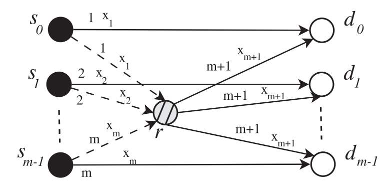
Symbol	Definition
NC-CC	Network Coded Cooperative Communications.
W	Total bandwidth available in the network
SNR_{uv}	The signal to noise ratio between nodes u and v
α_r	Amplification factor at relay node r
σ_v^2	Variance of background noise at node v
$\sigma^2_{z_v^{\mathrm{ANC}}}$	Variance of ANC noise at node v
h_{uv}	Effect of path-loss, shadowing, and fading from node u to node v
z_v	White Gaussian background noise at node v
z_v^{ANC}	ANC noise at node v
w_i	Weight assigned to session (s_i, d_i)
x_i	Signal transmitted by source s_i
y_{uv}	Signal transmitted by node u and received by node v
$y_{\mathcal{R}d_i}$	Signal transmitted by relay group $\mathcal{R}$ and received by node d_i
$y_{\mathcal{SR}d_i}$	Signal received by destination d_i that was originally transmitted by session group $\mathcal{S}$ and then retransmitted by relay group $\mathcal{R}$
h_{uv}	Effect of path-loss, shadowing, and fading from node u to node v
P_u	Transmission power at node u
$\mathcal{R}$	The set of all relay nodes in the network
$\mathcal{S}$	The set of all source nodes in the network
$\mathcal{R}_j$	A group of relay nodes
$\mathcal{S}_{s_i}^{\mathcal{R}_j}$	Group of sessions containing s_i and using relay group $\mathcal{R}_j$ for NC-CC

components. We show that this problem is NP-hard. Subsequently, we develop a highly competitive and efficient algorithm to solve this problem.

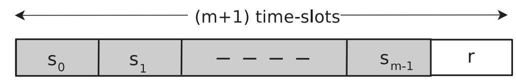
The remainder of this paper is organized as follows. In Section 2, we review state-of-the-art results on NC-CC when only a single relay node is employed. In Section 3, we study NC-CC with multiple sessions and multiple relay nodes. We develop formulas for the mutual information and achievable data rate of each session. In Section 4, we describe the session/relay grouping and matching problem in detail. We also show that this problem is NP-hard. In Section 5, we present an algorithm to this problem. Section 6 presents numerical results to demonstrate the performance and efficiency of the proposed algorithm. In Section 7, we discuss related work, and Section 8 concludes this paper. Table 1 lists all notation used in this paper.

2. Background

As a simple example, Fig. 1 shows a three-node relay channel for CC. In this example, source node s intends to transmit to destination node d and will exploit relay node r for possible performance improvement. Assume the time frame for transmission is divided into two time slots. We show time slot for each link in Fig. 1. That is, in the first time slot, s transmits to d, and is overheard by the relay node r. Relay node r then amplifies the received signal, and then retransmits the amplified signal in the second time slot. The destination node d can now combine the two copies of the same signal coming from two different paths. This cooperative relay channel in Fig. 1 can be treated as a single-input two-output complex Gaussian channel [15] and the achievable data



(a) m source nodes sharing one relay node.



(b) Structure of a time frame.

Fig. 2. NC-CC with a single relay node.

rate between s and d in this channel is given as follows:

$$C_{\mathrm{CC}}(s,r,d) = \frac{W}{2}\log_2\left(1+\mathrm{SNR}_{sd}+\frac{\mathrm{SNR}_{sr}\cdot\mathrm{SNR}_{rd}}{1+\mathrm{SNR}_{sr}+\mathrm{SNR}_{rd}}\right), \tag{1}$$

where SNR_{uv} is the signal-to-noise ratio at the receiver v when node u transmits, and is given by $\mathrm{SNR}_{uv} = \frac{|h_{uv}|^2 P_u}{\sigma_v^2}$, σ_v^2 is the variance of background noise at node v, h_{uv} is the gain of the channel from node u to node v, P_u is the power at which node u transmits signals, and W is the channel bandwidth.

As for comparison, when CC is not used, i.e., s transmits to d without using r (so-called direct transmission), the achievable rate from s to d is given as:

$$C_{\mathrm{D}}(s,\emptyset,d) = W\log_2(1+\mathrm{SNR}_{sd}), \tag{2}$$

where $\emptyset$ denotes that no relay node is used. It has been shown in [15] and [28] that CC has the potential to increase achievable rate over direct transmission, depending on the location of relay node and its channel statistics.

When there are m source-destination sessions sharing a single relay node (as shown in Fig. 2), one can employ NC to combine the signals from the m sources at the relay node and then forward the combined signal to all the destination nodes, i.e., NC-CC. Here, a time frame is divided into $(m+1)$ time slots (see Fig. 2b), with time slot i, $i \le m$, being used for transmission by source node s_{i-1}. Again, each of such transmissions is received by its corresponding destination node and overheard by the relay node. The relay node will then apply NC to combine all the m received analog signals. The combined signal x_m is then amplified and forwarded by the relay node to all destination nodes in the $(m+1)$th time slot. The achievable rate for a session under NC-CC is given by Sharma et al. [25]:

$$C_{\mathrm{NC\text{-}CC}}(s_i,\mathcal{S},r,d_i,) = \frac{W}{m+1}\log_2\left(1+\mathrm{SNR}_{s_id_i}+\frac{\mathrm{SNR}_{s_ir}\,\mathrm{SNR}_{rd_i}}{|\mathcal{S}|\frac{\sigma^2_{z_{d_i}^{\mathrm{ANC}}}}{\sigma^2_{d_i}}+\mathrm{SNR}_{rd_i}+\frac{\sigma^2_{z_{d_i}^{\mathrm{ANC}}}}{\sigma^2_{d_i}}\sum_{s_j\in\mathcal{S}}\mathrm{SNR}_{s_jr}}\right), \tag{3}$$

where $\mathcal{S} = \{s_0, s_1, \ldots, s_{m-1}\}$ is the set of all the source nodes, and $\sigma^2_{z_{d_i}^{\mathrm{ANC}}}$ is the noise at destination d_i due to ANC, and is given by

$$\sigma^2_{z_{d_i}^{\mathrm{ANC}}} = \sigma^2_{d_i}+(|\mathcal{S}|-1)\left(\alpha_r h_{rd_i}\right)^2\sigma_r^2+\sigma^2_{d_i}\sum_{s_j\in\mathcal{S}}^{s_j\neq s_i}\left(\frac{\alpha_r h_{s_jr}h_{rd_i}}{h_{s_jd_i}}\right)^2, \tag{4}$$

where α_r is the amplification factor for AF CC at relay node r and is given by

$$\alpha_r^2 = \frac{P_r}{|\mathcal{S}|\sigma_r^2+\sum_{s_i\in\mathcal{S}}P_{s_i}\ |h_{s_ir}|^2}. \tag{5}$$