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Sine entropy of uncertain set and its applications

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ABSTRACT

Entropy is used as a quantitative measurement of the information deficiency. In uncertain set theory, logarithm entropy for uncertain set has been proposed. However, it fails to measure the degree of uncertainty associated with some uncertain sets. Thus this paper will propose sine entropy for uncertain set as a supplement, and investigate its properties such as translation invariance and positive linearity. Besides, it proposes sine relative entropy for uncertain set, and gives its applications in portfolio selection and clustering.

1. Introduction

Entropy, a quantitative measure of the information, was first proposed for a random variable by Shannon [22] in the form of logarithm in 1948. Then quadratic entropy was presented by Vaida [23] as a supplement to measure the information deficiency of a random variable in 1968. As we know, given the variance and the expected value of a random variable, there are many probability distributions meeting with the constraints. In this case, Jaynes [12] proposed the maximum entropy principle, that is to choose the probability distribution with the maximum entropy.

In 1965, Zadeh [25] proposed a concept of fuzzy set via membership function to model the fuzzy event. Then fuzzy inference based on fuzzy set was widely applied in engineering and other areas. An application of fuzzy inference in spatial analysis was given by Arabacioglu [1], in which he employed Mamdami inference system to analyze the architectural space and showed that the system performs very well even with imprecise data. In 1968, Zadeh [26] introduced entropy to fuzzy set as a measure of the fuzziness. After that, the concept of entropy for fuzzy set was refined by De Luca and Termini [7] in 1972, and further studied by many researchers such as Pal and Bezdek [20], and Yager [24].

In order to deal with human uncertainty, many theories has been proposed such as Dempster-Shafer theory (Dempster [8], Shafer [21]). In 2007, Liu [14] proposed a branch of axiomatic mathematics to deal with human uncertainty, which is called uncertainty theory. Then Liu [15] proposed a concept of entropy for uncertain variable, which was further studied by Chen, Dai and their co-workers [3,5,6]. Uncertain variable is used to model the quantity in the status of uncertainty. In order to model the unsharp concepts, Liu [16] proposed a concept of uncertain set as a generalization of uncertain variable, and applied it to uncertain inference. In 2010, Gao, Gao and Ralescu [9] extended the uncertain inference rule to the case with multiple antecedents and multiple if-then rules. After that, Gao [10] balanced an inverted pendulum by using the uncertain controller. In addition, uncertain set has also been used to multi-valued logic. In 2011, Chen and Ralescu [4] gave a formula to calculate the truth value of an uncertain proposition, and Liu [17] gave an application of uncertain logic in modelling human language. After that, the concept of uncertain set was refined by Liu [19] in 2012.

Entropy for uncertain set has been proposed by Liu [17] as a measure of information deficiency in the form of logarithm function. However, it cannot measure the uncertainty associated with all uncertain set. Thus we will propose sine entropy for uncertain set as a supplement in this paper, and investigate its main properties such as translation invariance and positive linearity. The rest of this paper is structured as follows. The next section is intended to introduce some concepts of uncertain set and membership function. Then we

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introduce the concept of sine entropy in Section 3, and propose a formula to calculate sine entropy via inverse membership function in Section 4. After that, a concept of sine relative entropy is presented in Section 5, and its applications in portfolio selection and clustering are given in Sections 6 and 7, respectively. At last, we discuss the relationship between uncertainty theory, probability theory, and fuzzy theory for readers to better understand the paper in Section 8.

2. Preliminary

Uncertainty theory is a branch of axiomatic mathematics based on normality, duality, subadditivity and product axioms. The basic and essential definition in uncertainty theory is uncertain measure.

Definition 1. (Liu [14]) Let $\mathcal{L}$ be a σ-algebra on a nonempty set Γ. A set function $\mathcal{M}:\mathcal{L}\to[0,1]$ is called an uncertain measure if it satisfies the following axioms:

Axiom 1 : (Normality Axiom) $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ.
Axiom 2 : (Duality Axiom) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ.
Axiom 3 : (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\{\bigcup_{i=1}^{\infty}\Lambda_i\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Besides, the product uncertain measure on the product σ-algebre $\mathcal{L}$ is defined by Liu [15] as follows,
Axiom 4 : (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k=1, 2, \ldots$. Then the product uncertain measure $\mathcal{M}$ on the product σ-algebra $\mathcal{L}_1\times\mathcal{L}_2\times\cdots\times\mathcal{L}_k$ is an uncertain measure satisfying

$$\mathcal{M}\{\prod_{i=1}^{\infty}\Lambda_k\}=\bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k=1, 2, \ldots$, respectively.

Now, we introduce definitions of uncertain set and membership function as well as their operational law.

Definition 2. (Liu [19]) An uncertainty set is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to a collection of sets of real numbers, i.e., for any Borel set B of real numbers, the following two sets

$$\{\xi\subset B\}=\{\gamma\in\Gamma\mid\xi(\gamma)\subset B\},$$
$$\{B\subset\xi\}=\{\gamma\in\Gamma\mid B\subset\xi(\gamma)\}$$

are events.

The union and intersection of uncertain sets ξ and η are defined as $(\xi\cup\eta)(\gamma)=\xi(\gamma)\cup\eta(\gamma)$ and $(\xi\cap\eta)(\gamma)=\xi(\gamma)\cap\eta(\gamma)$ for every $\gamma\in\Gamma$, respectively. They are also uncertain sets.

Definition 3. (Liu [19]) An uncertain set ξ is said to have a membership function μ if the equations

$$\mathcal{M}\{\xi\subset B\}=\inf_{x\in B}\mu(x)$$
$$\mathcal{M}\{B\subset\xi\}=1-\sup_{x\in B^c}\mu(x)$$

holds for any Borel set B of real numbers.

Liu [16] proved that a real-valued function μ is a membership function if and only if $0\le\mu(x)\le1$. A membership function is said to be regular if there exists a point x_0 such that $\mu(x_0)=1$ and $\mu(x)$ is unimodal about the point x_0. Triangular uncertain set (a, b, c) is a type of frequently used uncertain sets with a membership function

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & \text{if} \quad a\le x\le b\\[2mm] \dfrac{x-c}{b-c}, & \text{if} \quad b\le x\le c\end{cases}$$

shown in Fig. 1. Trapezoidal uncertain set $\xi=(a, b, c, d)$ is another type of frequent used uncertain sets with a membership function

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & \text{if} \quad a\le x\le b\\[2mm] 1, & \text{if} \quad b\le x\le c\\[2mm] \dfrac{x-d}{c-d}, & \text{if} \quad c\le x\le d\end{cases}$$

shown in Fig. 2.

Based on membership function, a concept of entropy for uncertain set was proposed in the form of logarithm function.