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Short communication

Orthogonal projection method for DOA estimation in low-altitude environment based on signal subspace

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ABSTRACT

The rank-deficiency and subspace leakage caused by multipath effect are the main factors that lead to performance breakdown of direction of arrival (DOA) estimation in low-altitude environment. In this paper, we propose an orthogonal projection method based on signal subspace to overcome the negative effects of multipath. First, the signal covariance matrix is recovered to full-rank by forward and backward spatial smoothing (FBSS). Then, based on the least square technique, the signal subspace is used to establish the orthogonal projection matrix. Thereby the cross covariance matrices of signal and noise parts can be estimated and eliminated to modify the sample covariance matrix. Compared with the conventional methods that only dispose rank-deficiency, the proposed method has better performances in low-altitude environment. Besides, compared with the former orthogonal projection method based on steering matrix, this method reduces the computational complexity without iterative scheme. These conclusions are verified by simulations.

1. Introduction

The multipath effect (including specular reflection and diffuse reflection) in low-altitude environment will seriously degrade the performance of conventional direction of arrival (DOA) estimation algorithms such as multiple signal classification (MUSIC) [1] and estimation signal parameters via rotational invariance technique (ESPRIT) [2]. The specular reflection will lead to rank-deficiency and some directions may lose [3]. The diffuse reflection signals will be received as noise, thus the noise will be correlated with target signals to some extent. This will lead to the so-called subspace leakage problem, which means part of the true signal subspace resides in the sample noise subspace (and vice versa) [4]. Many methods have been proposed to solve the rank-deficiency problem from different aspects. There are methods such as forward and backward spatial smoothing (FBSS) [5], oblique projection [6] to recover the covariance matrix to full-rank. The [7] and [8] both adopt improved ML method to avoid the rank-deficiency problem. The [9] uses the spatial diversity of MIMO radar to avoid the targets glint caused by multipath. Based on sparse signal reconstruction method, [10] and [11] overcome the rank-deficiency problem without estimating the covariance matrixes. But the study of subspace leakage comparatively attracts less attention. To solve this problem, Steinwandt directly models leakage of noise subspace into signal subspace and then estimates the corresponding perturbation matrix [12]. [13] proposes an orthogonal projection method based on array steering matrix, however the DOA estimation procedure has to be repeated and the computational complexity is large.

In this paper, we propose a novel orthogonal projection method to improve the DOA estimation performance in low-altitude environment. The signal subspace, instead of the steering matrix, is used to establish the orthogonal projection matrix. Compared with the method of [13], the proposed method can reach the same accuracy with smaller computational complexity. The notations of $(\cdot)^{\mathrm{T}}$,$(\cdot)^{*}$ and $(\cdot)^{\mathrm{H}}$ denote the transpose, conjugation and the conjugation-transpose of the matrixes, respectively.

2. Signal model in low-altitude environment

The array geometry model in low-altitude environment is shown in Fig. 1.

Consider an uniform linear array (ULA) with M sensors, whose inter-element space is half-wavelength. Far field narrowband signals impinge on this array from the directions θ_k $(k=1,2,\cdots,K)$. $\boldsymbol{s}(t)=[s_1(t),s_2(t),\cdots,s_K(t)]^{\mathrm{T}}$ denotes the uncorrelated narrowband signal vector. The ULA receives both the direct and reflected signals. The steering matrix of the array is $\boldsymbol{A}=[\boldsymbol{a}_{d1}+\boldsymbol{a}_{r1},\cdots,\boldsymbol{a}_{dK}+\boldsymbol{a}_{rK}]$, and:

$$\boldsymbol{a}_{dk}(\theta_{dk})=\left[1,\ \mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_{dk}},\cdots,\mathrm{e}^{-\mathrm{j}\,\pi(M-1)\sin\theta_{dk}}\right]^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{a}_{rk}(\theta_{rk})=\delta_k\boldsymbol{a}_{dk}(\theta_{rk}) \tag{2}$$

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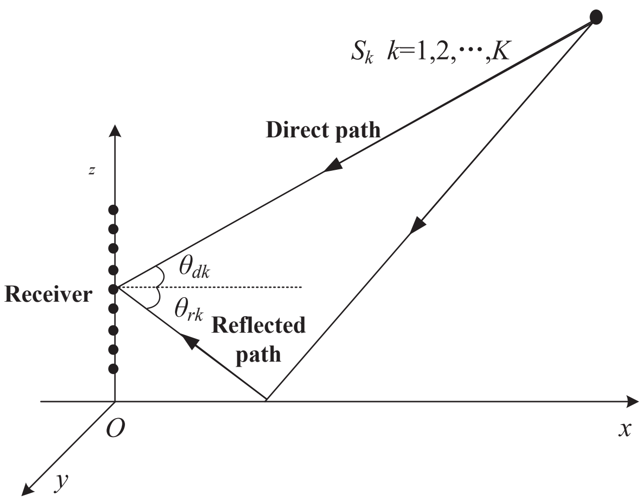

Fig. 1. The array geometry model in low-altitude environment.

where $\theta_{dk} = -\theta_{rk} = \theta_k$, δ_k is the multipath reflection coefficient. Thus:

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1) + \delta_1\boldsymbol{a}(-\theta_1), \ \cdots, \ \boldsymbol{a}(\theta_K) + \delta_K\boldsymbol{a}(-\theta_K)] \tag{3}$$

The output of receiver can be represented as:

$$\boldsymbol{y}(t) = \boldsymbol{As}(t) + \boldsymbol{n}(t) = \boldsymbol{Bx}(t) + \boldsymbol{n}(t) \tag{4}$$

where $\boldsymbol{n}(t)$ is complex colored noise, $\boldsymbol{B} = [\boldsymbol{a}(\theta_1),\boldsymbol{a}(-\theta_1),\cdots,\boldsymbol{a}(\theta_K),\boldsymbol{a}(-\theta_K)]_{N\times 2K}$, and $\boldsymbol{x}(t) = [s_1(t),\delta_1 s_1(t),\cdots,s_K(t),\delta_K s_K(t)]^{\mathrm{T}}_{2K\times 1}$. The covariance of $\boldsymbol{y}(t)$ is:

$$\boldsymbol{R} = E[\boldsymbol{y}(t)\boldsymbol{y}^{\mathrm{H}}(t)] = \boldsymbol{BR}_x\boldsymbol{B}^{\mathrm{H}} + \mathrm{E}[\boldsymbol{n}(t)\boldsymbol{n}^{\mathrm{H}}(t)] \tag{5}$$

Due to the multipath effect, $\boldsymbol{x}(t)$ contains coherent signals and the $\boldsymbol{R}_x$ in (5) is a rank-deficient matrix. If $\boldsymbol{R}$ is directly used for eigen decomposition, some angles will lose. Therefore, the methods such as FBSS-MUSIC, oblique projection can be used to solve the rank-deficiency problem. Taking FBSS as an example, we can divide $\boldsymbol{y}(t)$ into L forward overlapping vectors $\boldsymbol{y}_{fl}(t)$ and L backward overlapping vectors $\boldsymbol{y}_{bl}(t)$, and each of them contains m antennas:

$$\boldsymbol{y}_{fl}(t) = \boldsymbol{F}_l\boldsymbol{y}(t) = \boldsymbol{B}_m\boldsymbol{D}^{l-1}\boldsymbol{x}(t) + \boldsymbol{n}_l(t) \tag{6}$$

$$\boldsymbol{y}_{bl}(t) = \boldsymbol{F}_l\boldsymbol{J}\boldsymbol{y}^{*}(t) \tag{7}$$

where $\boldsymbol{F}_l = [0_{m\times(l-1)}\ \boldsymbol{I}_m\ 0_{m\times(M-m-l+1)}]$, $l = 1,2,\cdots,L$. $\boldsymbol{B}_m$ is the first m rows of $\boldsymbol{B}$ and is an $m \times m$ exchange matrix with 1 on its anti-diagonal and 0 elsewhere. Besides:

$$\boldsymbol{D} = \mathrm{diag}[\mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_1},\mathrm{e}^{\mathrm{j}\pi\sin\theta_1},\cdots,\mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_K},\mathrm{e}^{\mathrm{j}\,\pi\sin\theta_K}] \tag{8}$$

The $(L-l+1)$ th backward vector can be expressed by the l th forward vector as:

$$\boldsymbol{y}_{b(L-l+1)}(t) = \boldsymbol{J}\boldsymbol{y}_{fl}^{*}(t) \tag{9}$$

Suppose the number of snapshots is $\boldsymbol{N}$. The covariance of $\boldsymbol{y}_{fl}(t)$ and $\boldsymbol{y}_{b(L-l+1)}(t)$ can be estimated respectively:

$$\widehat{\boldsymbol{R}}_{fl} = \frac{1}{N}\sum_{t=1}^{N}\ [\boldsymbol{y}_{fl}(t)\boldsymbol{y}_{fl}^{\mathrm{H}}(t)] = \boldsymbol{B}_m\boldsymbol{D}^{l-1}\widehat{\boldsymbol{R}}_x(\boldsymbol{B}_m\boldsymbol{D}^{l-1})^{\mathrm{H}} + \widehat{\boldsymbol{R}}_{\mathrm{n}}^{fl} + \widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl} + \widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl} \tag{10}$$

$$\begin{aligned}\widehat{\boldsymbol{R}}_{b(L-l+1)} &= \frac{1}{N}\sum_{t=1}^{N}\ [\boldsymbol{y}_{b(L-l+1)}(t)\boldsymbol{y}_{b(L-l+1)}^{\mathrm{H}}(t)]\\ &= \boldsymbol{J}\,[\boldsymbol{B}_m\boldsymbol{D}^{l-1}\widehat{\boldsymbol{R}}_x(\boldsymbol{B}_m\boldsymbol{D}^{l-1})^{\mathrm{H}} + \widehat{\boldsymbol{R}}_{\mathrm{n}}^{fl} + \widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl} + \widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl}]^{*}\boldsymbol{J}\end{aligned} \tag{11}$$

Hence, the total smoothing matrix can be expressed as:

$$\overline{\boldsymbol{R}} = \frac{1}{L}\sum_{l=1}^{L}\ \overline{\boldsymbol{R}}_l = \frac{1}{2L}\sum_{l=1}^{L}\ [\widehat{\boldsymbol{R}}_{fl} + \boldsymbol{J}(\widehat{\boldsymbol{R}}_{fl})^{*}\boldsymbol{J}] \tag{12}$$

The amount of coherent sources that can be identified depends on the method that is used to solve the rank-deficiency problem. In this paper, we use FBSS to solve the problem. For an array with M elements, the maximum coherent sources that the proposed algorithm can identify is $2M/3$, which has been proved in literature [5].

3. Orthogonal projection methods to solve subspace leakage

3.1. Orthogonal projection method based on steering matrix

The signal and noise are partially correlated, and the actual number of snapshots is finite. Thus the $\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl}$ and $\widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl}$ $(l = 1,2,\cdots,L)$ may have significant values, which may decrease the DOA estimation performance largely. In order to remove these terms, we employ the least square technique with (6) to estimate the source signal $\boldsymbol{x}$ as:

$$\widehat{\boldsymbol{x}} = [(\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1})^{\mathrm{H}}\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1}]^{-1}(\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1})^{\mathrm{H}}\boldsymbol{y}_{fl}(t) \tag{13}$$

where $\widehat{\boldsymbol{B}}_m$ is the estimator of $\boldsymbol{B}_m$ and the l th forward noise vector can be estimated as:

$$\widehat{\boldsymbol{n}}_{fl}(t) = \boldsymbol{y}_{fl}(t) - \widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1}\widehat{\boldsymbol{x}} = \boldsymbol{y}_{fl}(t) - \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}_m^{\mathrm{H}}\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}_m^{\mathrm{H}}\boldsymbol{y}_{fl}(t) = \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}\boldsymbol{y}_{fl}(t) \tag{14}$$

where $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp} = \boldsymbol{I}_m - \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$, and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m} = \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}_m^{\mathrm{H}}\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}_m^{\mathrm{H}}$. Therefore $\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl}$ and $\widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl}$ can be estimated as:

$$\begin{cases}\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl} = \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}\\ \widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl} = \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}\end{cases} \tag{15}$$

In reality, $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$ cannot be known at first, so [13] adopts a two-step scheme to estimate $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$. In the first step, the DOAs are estimated without eliminating $\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl}$ and $\widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl}$. In the second step, the steering matrix $\widehat{\boldsymbol{B}}_m$, together with $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}^{\perp}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$, is obtained using the estimated DOAs, so that the $\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl}$ and $\widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl}$ can be estimated and eliminated. Therefore, the DOAs can be estimated more precisely.

3.2. Improved orthogonal projection method based on signal subspace

The method based on steering matrix has to estimate the DOAs in ahead, which brings extra computational complexity to establish space spectrum and search peaks. To reduce the computational complexity, we propose a novel orthogonal projection method based on signal subspace. The eigen decomposition of $\overline{\boldsymbol{R}}$ is:

$$\overline{\boldsymbol{R}} = \widehat{\boldsymbol{U}}_{\mathrm{s}}\widehat{\boldsymbol{\Sigma}}_{\mathrm{s}}\widehat{\boldsymbol{U}}_{\mathrm{s}}^{\mathrm{H}} + \widehat{\boldsymbol{U}}_{\mathrm{n}}\widehat{\boldsymbol{\Sigma}}_{\mathrm{n}}\widehat{\boldsymbol{U}}_{\mathrm{n}}^{\mathrm{H}} \tag{16}$$

The diagonal matrix $\widehat{\boldsymbol{\Sigma}}_{\mathrm{s}}$ and $\widehat{\boldsymbol{\Sigma}}_{\mathrm{n}}$ are consisted of the largest K eigenvalues and the rest m-K eigenvalues respectively. $\widehat{\boldsymbol{U}}_{\mathrm{s}}$ and $\widehat{\boldsymbol{U}}_{\mathrm{n}}$ are the signal subspace and noise subspace respectively. The space spanned by the vectors of $\widehat{\boldsymbol{U}}_{\mathrm{s}}$ and the space spanned by the vectors of $\boldsymbol{B}_m$ are the same, namely:

$$\mathrm{span}\{\boldsymbol{u}_1,\boldsymbol{u}_2,\cdots,\boldsymbol{u}_K\} = \mathrm{span}\{\boldsymbol{b}_1,\boldsymbol{b}_2,\cdots,\boldsymbol{b}_K\} \tag{17}$$

There exists a $K \times K$ invertible matrix $\boldsymbol{V}$ which satisfies the equation of $\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V} = \boldsymbol{B}_m$, then the following equations can be obtained:

$$\begin{aligned}\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m} &= \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}_m^{\mathrm{H}}\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}_m^{\mathrm{H}} = \widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V}[(\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V})^{\mathrm{H}}\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V}]^{-1}(\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V})^{\mathrm{H}} = \widehat{\boldsymbol{U}}_{\mathrm{s}}(\widehat{\boldsymbol{U}}_{\mathrm{s}}^{\mathrm{H}}\widehat{\boldsymbol{U}}_{\mathrm{s}})^{-1}\widehat{\boldsymbol{U}}_{\mathrm{s}}^{\mathrm{H}}\\ &= \widehat{\boldsymbol{P}}_{\mathrm{s}}\end{aligned} \tag{18}$$

So $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$ can be replaced with $\widehat{\boldsymbol{P}}_{\mathrm{s}}$ to perform the orthogonal projection in (15):

$$\begin{cases}\widehat{\boldsymbol{R}}_{\mathrm{sn}}^{fl} = \widehat{\boldsymbol{P}}_{\mathrm{s}}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\mathrm{s}}^{\perp}\\ \widehat{\boldsymbol{R}}_{\mathrm{ns}}^{fl} = \widehat{\boldsymbol{P}}_{\mathrm{s}}^{\perp}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\mathrm{s}}\end{cases} \tag{19}$$